Name: KEY **Tribe:**

CHALLENGE 1

Integer Notation

Pitch-classes	Integer notation	Pitch-classes	Integer notation
D#	3	**G#**	8
Gb	6	**F**	5
Ab	8	**Bb**	10
C#	1	**E**	4
F#	6	**Db**	1
A	9	**G**	7

Mod 12 Arithmetic

1. Write the equivalent integer in *mod 12*.

21	**17**	**12**	**14**	**29**	**15**	**13**	**18**	**25**	**16**
9	5	0	2	5	3	1	6	1	4

2. Perform the calculations and write the results in *mod 12*.

7 + 6	**10 + 9**	**9 + 8**	**5 + 11**	**11 + 4**
1	7	5	4	3
2 – 8	**6 – 11**	**4 – 11**	**8 – 10**	**1 – 9**
6	7	5	10	4

Ordered vs. Unordered Interval-Classes

	Ordered	**Unordered**
G# → Bb	10 – 8 = 2	2
Db → F#	6 – 1 = 5	5
B → E	4 – 11 = -7 = 5	5
G → F#	6 – 7 = -1 = 11	1
D → A	9 – 2 = 7	5
Bb → Gb	6 – 10 = -4 = 8	4
F → Db	1 – 5 = -4 = 8	4

Rotation and Normal Form/Order

1. Convert each set into integer notation.
2. Rearrange into ascending order.
3. Calculate all possible orderings through rotation.
4. Calculate normal form.

Set: **E-Bb-Db-F**

1. 4, 10, 1, 5
2. 1, 4, 5, 10
3. 1, 4, 5, 10 (10 – 1 = 9)
   4, 5, 10, 1 (13 – 4 = 9)
   5, 10, 1, 4 (16 – 5 = 11)
   10, 1, 4, 5 (17 – 10 = 7)
4. Normal form = [10,1,4,5]

Set: **G#-C#-D-E-G**

1. 8, 1, 2, 4, 7
2. 1, 2, 4, 7, 8
3. **1, 2, 4, 7, 8 (8 – 1 = 7)**
   2, 4, 7, 8, 1 (13 – 2 = 11)
   4, 7, 8, 1, 2 (14 – 4 = 10)
   7, 8, 1, 2, 4 (16 – 7 = 9)
   8, 1, 2, 4, 7 (19 – 8 = 11)
4. Normal form = [1,2,4,7,8]

Name: KEY **Tribe:**

CHALLENGE 2

Transposition

- Determine whether the pairs of normal form sets are transpositionally equivalent. If yes, provide the transpositional operator.

Normal form sets	Transpositionally equivalent?	Index number (T_N)
[2,4,7] [9,10,0]	No	N/A
[8,10,11,1] [2,4,5,7]	Yes	T_6

a)
	9	10	0
-	2	4	7
	7	6	5

b)
	2	4	5	7
-	8	10	11	1
	6	6	6	6

- Perform the required transposition on the following normal form set.

Normal form sets	Transposition
T_{10} [0,2,4,5,8]	[10, 0, 2, 3, 6]

b)
	0	2	4	5	8
+	10	10	10	10	10
	10	0	2	3	6

Name: KEY **Tribe:**

CHALLENGE 3

Inversion

- Determine whether the following normal form sets are inversionally equivalent. If yes, provide the index number.

Normal form sets	Inversionally equivalent?	Index number ($T_N I$)
[1,2,3,7] [10,2,3,4]	Yes	$T_5 I$

AIS for [1,2,3,7]: 1,1,4

AIS for [10,2,3,4]: 4,1,1

1 + 4 = 5
2 + 3 = 5
3 + 2 = 5
7 + 10 = 17 = 5

- Perform the required inversion/transposition on the following normal form set.

Normal form set	Inversion/transposition
$T_{11}I$ [1,3,6,9]	[2,5,8,10]

Original: [1,3,6,9]

Inversion: 11,9,6,3

Normal form: [3,6,9,11]

3, 6, 9, 11 **11 – 3 = 8**
6, 9, 11, 3 3 – 6 = 9
9, 11, 3, 6 6 – 9 = 9
11, 3, 6, 9 9 – 11 = 10

	3	6	9	11
+	11	11	11	11
	2	5	8	10

Prime Form

Provide the prime form for the following normal form set.

1. (Put set in normal form.)
2. Transpose the set so that the first element is 0.
3. Invert the original, normal-form set and repeat steps 1 and 2.
4. Compare the results of step 2 and step 3; whichever is more packed to the left is prime form.

Normal form	Step two	Step three			Prime form
		Inversion of original	Normal form	Transposition	
[11,2,6]	[0,3,7]	[1,10,6]	[6,10,1]	[0,4,7]	(037)

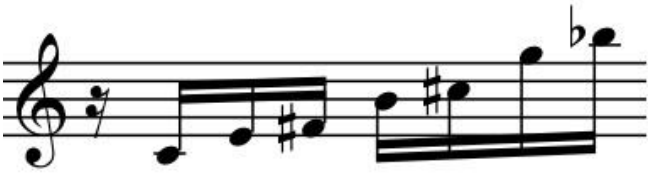
Bonus: What common musical structure is represented by the prime form of this set?
The minor triad (Note that this prime form represents both the minor triad and the major triad.)

Name: KEY **Tribe:**

CHALLENGE 4

Webern, *Five Movements for String Quartet,* IV, m. 6 (2nd Violin)

Convert the following set to integer notation, put the set in normal form, and calculate the prime form of the set.



[NOTE: Show additional work throughout these exercises.]

Integer notation: 0, 4, 6, 11, 1, 7, 10

Normal form: [10,11,0,1,4,6,7]
Transposed to begin on 0: (0123689)

Inversion of original, normal-form set: 2, 1, 0, 11, 8, 6, 5
Normal form of inversion: [11,0,1,2,5,6,8]
Transposed to begin on 0: (0123679)

Compare (0123689) with (0123679). The first five entries (and the last entry) are the same in both cases. The second-to-last entry breaks the tie: 7 is smaller than 8, so **(0123679) is the prime form**.

Interval-Class Vector

Calculate the interval-class vector for the set whose prime form you just found.

1. Calculate the intervals formed with the first note and each subsequent note in set.
2. Repeat the procedure with all of the notes except for the last one.
3. Add up all of the occurrences of each type of interval.
4. Show results in this format: [012111]. (In this example, there are no instances of ic1; there is one instance of ic2; etc.)

Prime form	**ICV**
(0123679)	[434343]

See work shown on next page.

	1	**2**	**3**	**4**	**5**	**6**
0123679	1	1	2		1	1
123679	1	1		1	1	1
23679	1			1	2	
3679			1	1		1
679	1		1			
79		1				
TOTAL:	**4**	**3**	**4**	**3**	**4**	**3**

ICV: [434343]

Name: KEY **Tribe:**

CHALLENGE 5

Prime Form

- Find the prime form of the normal form set [9,10,11,3]. Show your work.

1. (Put set in normal form.)
2. Transpose the set so that the first element is 0.
3. Invert the original, normal-form set and repeat steps 1 and 2.
4. Compare the results of step 2 and step 3; whichever is more packed to the left is prime form.

Normal form	Step two	Step three			Prime form
		Inversion of original	Normal form	Transposition	
[9,10,11,3]	[0,1,2,6]	[3,2,1,9]	[9,1,2,3]	[0,4,5,6]	(0126)

Transposition

- Determine whether the pair of normal form sets is transpositionally equivalent.
  If yes, provide the transpositional operator. Show your work.

Normal form sets	Transpositionally equivalent?	Index number (T_N)
[10,1,2,5] [5,8,9,0]	Yes	T_7

AIS for [10,1,2,5]: 3,1,3
AIS for [5,8,9,0]: 3,1,3

$$\begin{array}{rcccc} & 5 & 8 & 9 & 0 \\ - & 10 & 1 & 2 & 5 \\ \hline & 7 & 7 & 7 & 7 \end{array}$$

Inversion

- Perform the required inversion/transposition on the following normal form set.
  Show your work.

Normal form set	Inversion/transposition
T_5I [0,2,5]	[0,3,5]

Original: [0,2,5]
Inversion: 0,10,7
Normal form: [7,10,0]
Transposition: [0,3,5]

$$\begin{array}{rccc} & 7 & 10 & 0 \\ + & 5 & 5 & 5 \\ \hline & 0 & 3 & 5 \end{array}$$

Name: KEY **Tribe:**

CHALLENGE 6 (Requires a List of Set Classes)

Invariant Tones under Transposition

- For set (0247), how many pitch classes remain invariant under T_7?

  2

- What transpositions produce the maximum number of invariant pitch classes for set 4-12?

  T_3/T_9 and T_6

  ICV: [112101]

  Recall that the number of invariants under a given transposition for interval classes 1–5 and 7–10 is equal to the number of occurrences in the set's interval-class vector. However, the number of invariants under a given transposition for interval class 6 is equal to *twice* the number of occurrences in the set's interval-class vector.

Invariant Tones under Inversion

- Complete a summation square for the following set (0247), give the index vector, and answer the accompanying questions.

Summation square:

	0	**2**	**4**	**7**
0	0	2	4	7
2	2	4	6	9
4	4	6	8	11
7	7	9	11	2

Index vector: <1,0,3,0,3,0,2,2,1,2,0,2>

- What inversions produce the maximum number of invariants?

  T_2I, T_4I

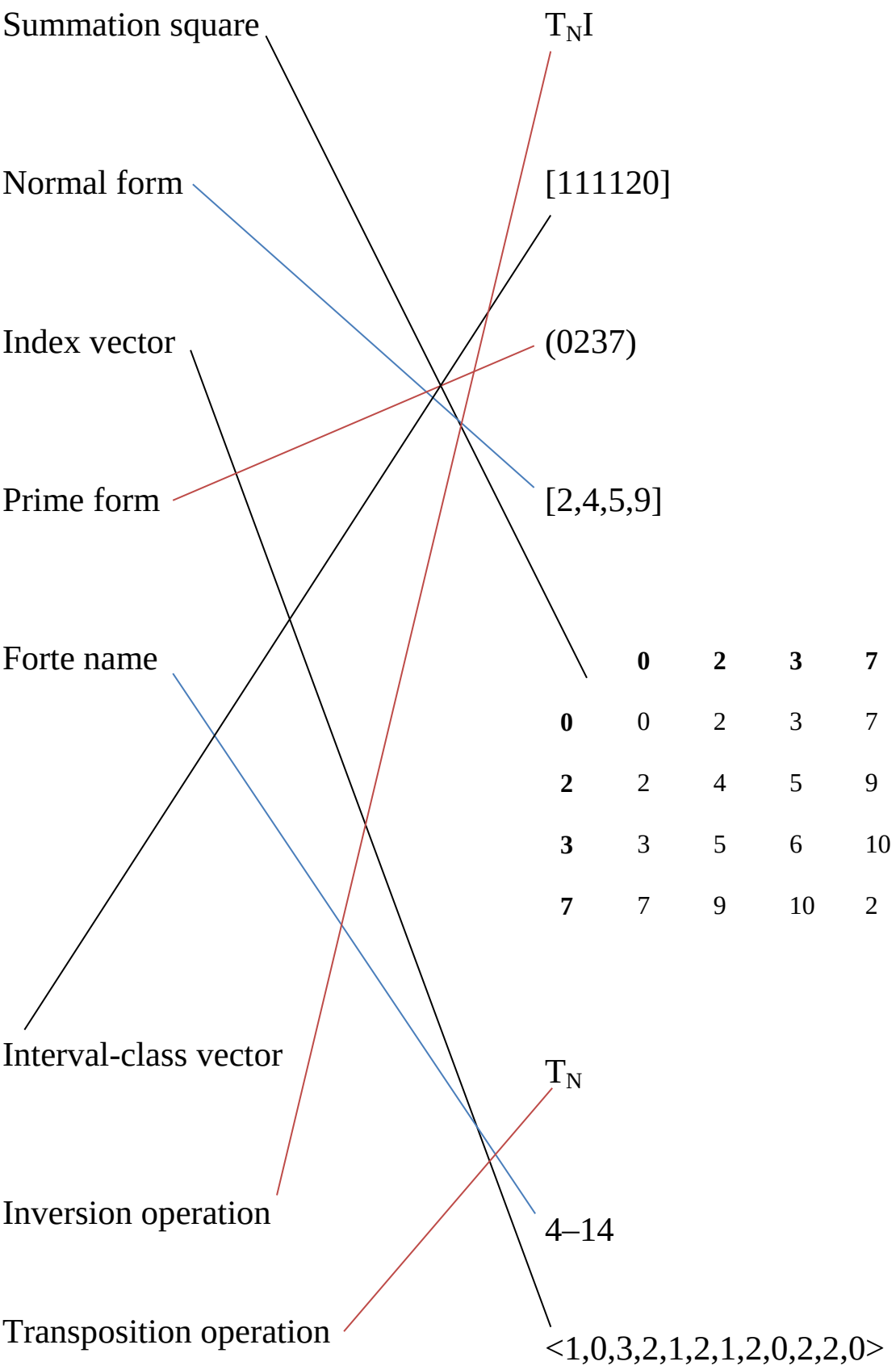
- How many invariants are produced by these inversions?

  3 invariants each

- What inversions produce no invariants?

  T_1I, T_3I, T_5I, $T_{10}I$

Reward Challenge: Match each term in the left-hand column with its representation in the right-hand column.



	0	**2**	**3**	**7**
0	0	2	3	7
2	2	4	5	9
3	3	5	6	10
7	7	9	10	2

Name: KEY **Tribe:**

CHALLENGE 7

Transposition

- Perform the indicated transposition operation. Show your work.

T_5 [8,9,11,1] = **[1,2,4,6]**

$$\begin{array}{cccc} 8 & 9 & 11 & 1 \\ + \quad 5 & 5 & 5 & 5 \\ \hline 1 & 2 & 4 & 6 \end{array}$$

Inversion

- Determine whether the following pair of normal form sets is inversionally equivalent. If yes, provide the index number. Show your work.

Normal form sets	Inversionally equivalent?	Index number ($T_N I$)
[4,5,7,9] [6,8,10,11]	Yes	$T_3 I$

AIS for [4,5,7,9]: 1,2,2

AIS for [6,8,10,11]: 2,2,1

4 + 11 = 15 = 3

5 + 10 = 15 = 3

7 + 8 = 15 = 3

9 + 6 = 15 = 3

Prime Form

- Find the prime form of the normal form set [8,9,11,1]. Show your work.

1. (Put set in normal form.)
2. Transpose the set so that the first element is 0.
3. Invert the original, normal-form set and repeat steps 1 and 2.
4. Compare the results of step 2 and step 3; whichever is more packed to the left is prime form.

Normal form	Step two	Step three			Prime form
		Inversion of original	Normal form	Transposition	
[8,9,11,1]	[0,1,3,5]	[4,3,1,11]	[11,1,3,4]	[0,2,4,5]	(0135)

Invariant Tones under Transposition

- What transpositions produce two invariant pitch classes for set (0135)?

ICV: [121110]

T_2 and T_{10}

Invariant Tones under Inversion

- Complete a summation square for set (0135) and give its index vector.

Summation square:

	0	**1**	**3**	**5**
0	0	1	3	5
1	1	2	4	6
3	3	4	6	8
5	5	6	8	10

Index vector: <1,2,1,2,2,2,3,0,2,0,1,0>