

Cycling Through Polyrhythms

Nick Rissman

The formal study of polyrhythms (2:3, 3:4, 3:4:5 etc.) often begins in the typical four-semester undergraduate theory sequence. Polyphonic instrumentalists are, no doubt, introduced to these rhythms earlier. Commonly, the rhythms are taught with the aid of subdivisions, the least common denominators shared by the separate parts of a polyrhythm. For example, the polyrhythm 3:2 has six subdivisions (the product of its two separate parts). Typically, this information is presented as a linear index¹ (Ex. 1a), or as a series of counts² (Ex. 1b) indicating the exact moments each part is to be articulated:

Example 1a

Example 1b

Example 1

This paper suggests teaching and understanding polyrhythms in a somewhat different manner: looping the subdivisions around themselves so that they form repetitive cycles and, in the process, mimic the motions of rotating gears rather than a linear index or series of counts. One advantage is that oral counting of the subdivisions

¹A linear index is used by S. Schick in “Developing and Interpretive Context: Learning from Brian Ferneyhough’s *Bone Alphabet*”, *Perspectives of New Music* 32.1:132-53 (1994). The index is applied to an impressive 10:12 subdivided polyrhythm and is perhaps the only solution, given the subdivided texture of the passage.

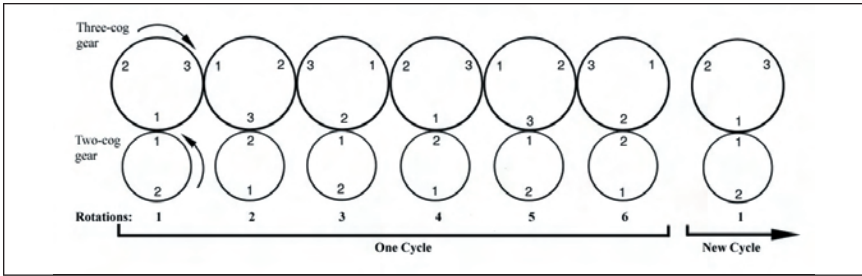
²Robert Starer offers a series of six “preliminary exercises”—various two-part polyrhythm drills—in *Rhythmic Training* (New York: MCA, 1969). Meter and rhythm are imposed on four abstract patterns: 2:3 (3:2), 3:4 (4:3), 2:5, and 3:5. This method encourages the counting of subdivisions.

is unnecessary and thus, the polyrhythm is more easily integrated into the composition at hand. (It would be difficult, for instance, to simultaneously count the polyrhythms' subdivisions and the actual metrical units in Exx. 1a and 1b). A second advantage is that one of the "gears" always coincides with the metrical unit; this allows the polyrhythm to be prepared a beat or two prior to its occurrence, something that is of particular value to the ensemble performer. The process is easily demonstrated, learned, and becomes a valuable reference tool while motivating students to explore ever more complex polyrhythms.

I. THEORY

A large gear and a small gear rotate at different rates based on their relative circumferences, and yet are joined at their cogs. A "cog" represents each gear's *lowest common denominator*. Example 2 illustrates the meshing of a two-cogged gear with a three-cogged gear, a ratio of 2:3. Note that six rotations are necessary before the two gears return to their starting position. This "cycle" of rotations (6) is the *least common multiple* of the two gears, and is the product of the two gears' cogs ($2 \times 3 = 6$). As ratios increase in size, it is important to be certain that the product of the two parts has in fact been reduced to the least common multiple, lest one's gears end up twice as large as necessary to complete a cycle. To assure this, keep two formulas in mind:

1. The highest common factor = the least common multiple for each of the two parts.
2. Where x and y are different prime numbers: xy = The least common multiple. The product of 10:12, for example, is 120. However, 120 can be further reduced to 60. Thus, 60 rotations equal one cycle of 10:12.



Example 2

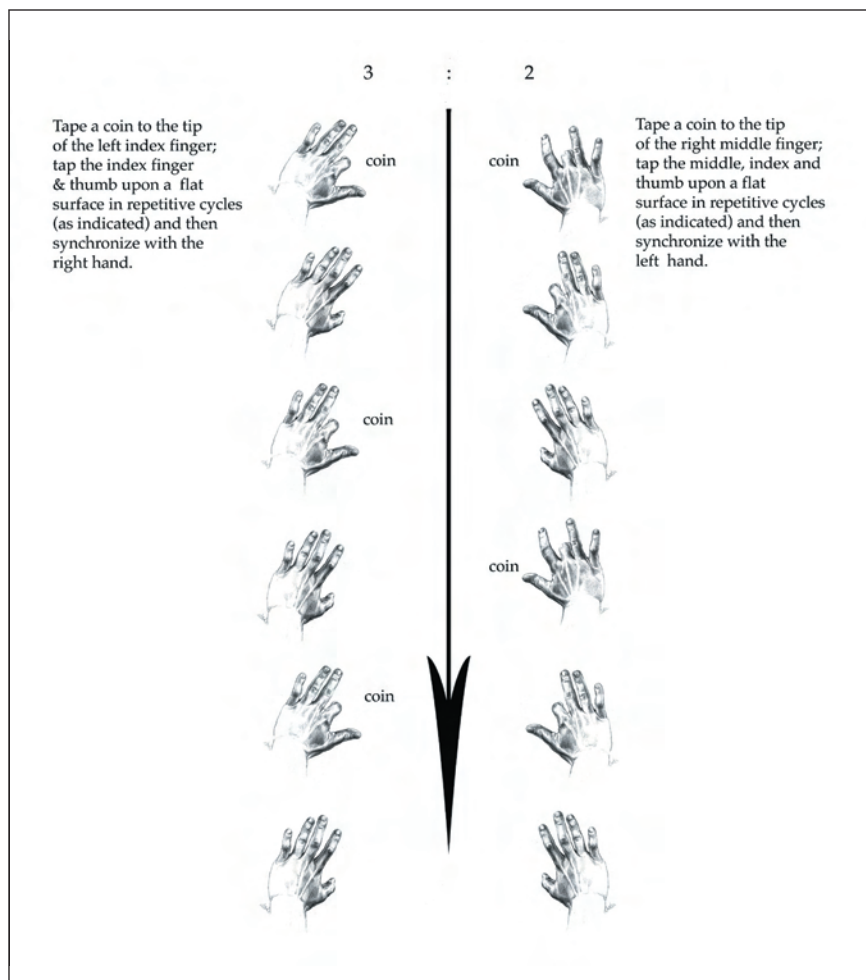
Returning to our two gears, imagine them unwound from their circumferences into lines, then laid flat with their cogs side by side (Ex. 3). The smaller gear, of course, needs to rotate or, in this case, repeat itself three times for every two repetitions of the large gear before returning to the starting position (Ex. 4). This process of “unwinding” can be made audible by taping a single coin (in the case, for example, of 2:3) to the tip of one hand’s middle finger and the other hand’s index finger. Tap both hands’ fingers simultaneously and consecutively *inward* from the “coined-fingers”--through each intervening finger--towards the thumbs in repetitive cycles. The *sound* (“coined-fingers”) of the polyrhythm emerges automatically, as does the *feel* of each of the rotations (including the “non-coined fingers”). Students master the process surprisingly quickly, and since important information is obtained both by *sound* and *touch*, oral counting of the subdivisions is unnecessary, a feature that will be useful when the process is applied to actual music passages.

Example 3

1 2
 [/ /] Two "cogs" in this gear
 [/ / /] Three "cogs" in this gear
 1 2 3

Example 4

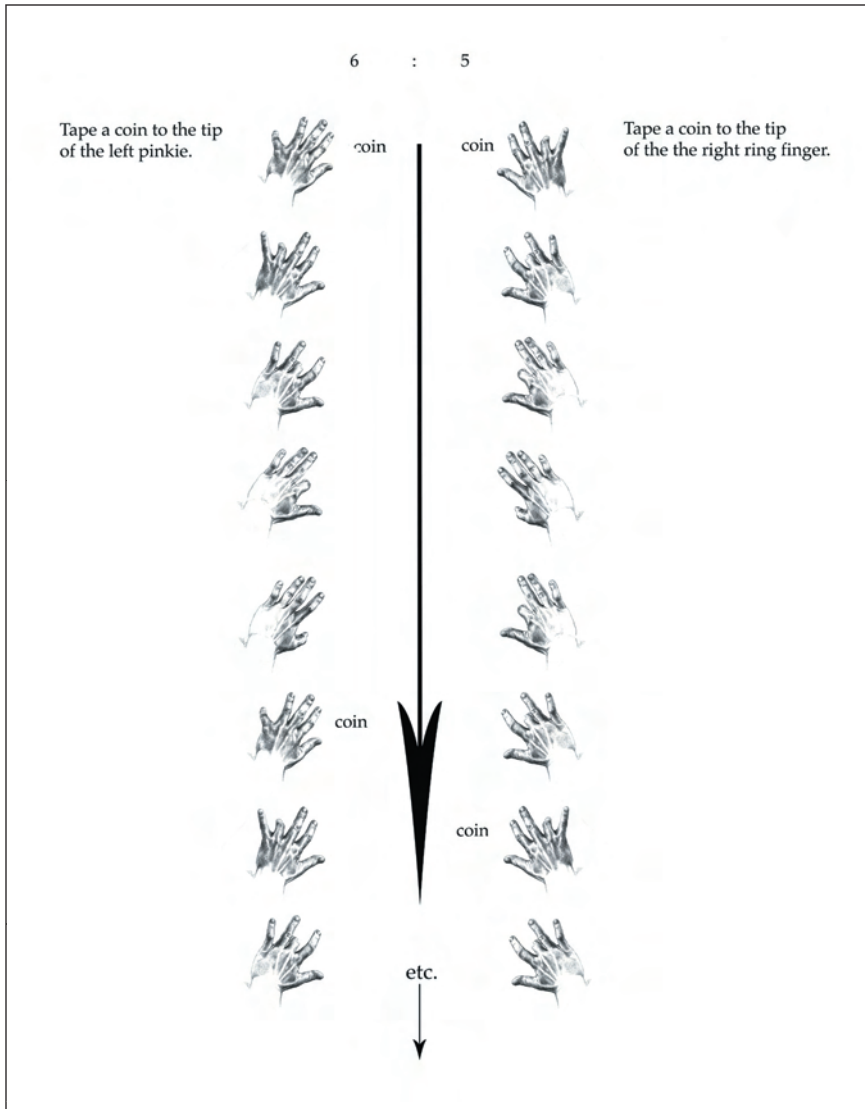
1 2 1 2 1 2
 [/ /][/ /][/ /]
 [/ /][/ /][/ /]
 1 2 3 1 2 3
 |-----one cycle-----|



Example 5. 2:3 (one cycle)

Any pattern from 5:4 downward may be performed in this manner. Additionally, the procedure may be applied to ratios greater than 5:4, as Ex. 6 illustrates. All that is required is a logical, *repetitive* sequence of fingers so that the mind can forget what the hands are doing.³

³It is important to stress that this procedure is intended for the undergraduate student, not necessarily the undergraduate piano student. In the past, some of the more innovative solutions to complex polyrhythms have come from my instrumentalists and vocalists, rather than from the keyboardists.



Example 6. 6:5 (a partial cycle). Note the right hand.

II. Application

While polyrhythms may exist as abstract patterns without the constraints of meter, it is important to understand them in a metrical context. There are four types: 1) A *Sub-metrical* polyrhythm nests within a portion of the beat (Ex. 7a); 2) A *Metrical* polyrhythm nests within the beat (Ex. 7b); 3) A *Super-metrical, coinciding* polyrhythm

is longer than the duration of the beat, with one of its parts coinciding with the beat (Ex. 7c); 4) A *Super-metrical, non-coinciding* polyrhythm is longer than the duration of the beat, with *none* of its parts coinciding with the beat (Ex. 7d).



Example 7a: *To Wake the Dead* (II. "Riverrun") by Stephen Albert.
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Example 7b: Sonata No. 9 for Piano, Op. 103 (mvt. IV) by Serge Prokofiev.
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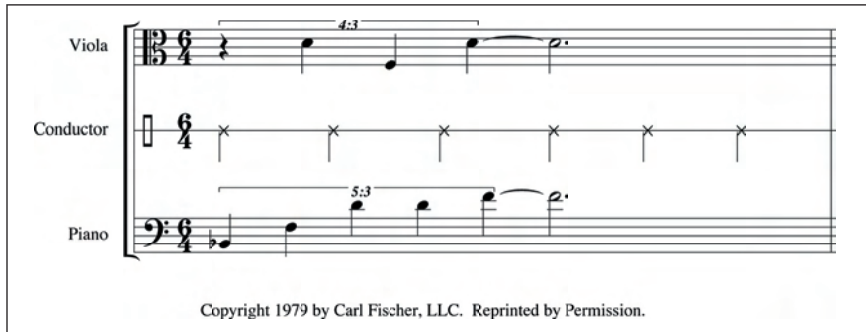
Example 7c: Sonata (for Flute, Oboe, Cello and Harpsichord) (mvt. III) by Elliott Carter.
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Example 7d

Examples 7a, 7b, 7c, and 7d.

Sub-metrical and metrical types are more easily mastered than the super-metrical variety because the meter itself is not a factor. However, super-metrical polyrhythms involve the beat, which in essence becomes a third part (Ex. 8).



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Example 8. To Wake the Dead (VII. "Passing Out") by Stephen Albert

To illustrate the application of our "coined-finger" method to an actual musical passage, let us first turn to a *metrical* example (Ex. 9). Example 10 demonstrates the "coined-finger" method applied to a *super-metrical, coinciding* example.



Example 9. Brahms. Concerto No. 1 in D Minor for Piano and Orchestra, mvmt. II, m. 87

The image shows a musical score for a piece titled "Passing Out" by Stephen Albert. The score is for a conductor, soprano, and piano. The tempo is marked as $\text{♩} \sim 63-72$. The conductor's part is a single line with a 5:3 polyrhythm indicated by a bracket and the numbers 5 and 3. The soprano part has lyrics "First we" and "pass through". The piano part has lyrics "pass through" and "through". The score is written in 4/4 time. The conductor's part is a single line with a 5:3 polyrhythm indicated by a bracket and the numbers 5 and 3. The soprano part has lyrics "First we" and "pass through". The piano part has lyrics "pass through" and "through". The score is written in 4/4 time.

From *To Wake the Land* (VII, "Passing Out") by Stephen Albert. Copyright 1979 by Carl Fischer, LLC. Reprinted by Permission.

Example 10. Because the quintuplet in m. 2 is "super-metrical," it forms a 5:3 polyrhythm with the conductor's beat. Cycling facilitates perfect alignment of such "-uplets." (Note how the fifth and sixth beats of m. 1 are used to prepare the polyrhythm.)

Perhaps the greatest challenge in Example 10 is context. Since the conductor's beat is one part of the polyrhythm (due to the slow tempo), it is crucial that the pattern be properly aligned with the baton. Therefore, tape a coin to the tip of the right hand's "pinkie" and the left hand's middle finger, then tap as indicated in Ex. 10. It is useful to observe that the right hand (with the coin taped to the pinkie) will repeat three times for every five repetitions of the left hand (with the coin taped to the middle finger). Thus, the right hand is the 3 in the ratio 5:3, and it coincides with the beat. Example 10 also illustrates how the polyrhythm can be prepared a few beats before its occurrence, to guarantee proper alignment with the baton. Example 11 is an alternate presentation (using piano-fingerings) of Ex. 10, and is better suited to our purposes as we continue.

The musical score for Example 11 consists of four staves, each with a 6/4 time signature. Above the staves are index numbers for subdivisions. The R.H. staff has a sequence of notes with a '5' above them. The Conductor staff has a sequence of notes with a '3' above them. The Piano staff has a sequence of notes with a '5' above them. The L.H. staff has a sequence of notes with a '3' above them. The score is enclosed in a box.

Example 11

More problematic is the *super-metrical, non-coinciding* type. In Ex. 8, for instance, neither the viola (4) nor the piano (5) coincides with the the conductor's beat (3), resulting in a 3:4:5 polyrhythm. To obtain the least common multiple, multiply the three parts by one another. The result is sixty rotations per cycle. Example 12 demonstrates a modified version of the "coined-finger" approach to this polyrhythm: Here, the greatest benefit to this approach is that, like gears, it is cyclic. In contrast, an index of the subdivisions would be linear; a rather unwieldy collection of integers that are difficult to count because they are non-cyclic (Ex. 13).

Polyrhythms are all around us: the sounds of our steps when walking with a friend; the simultaneous sounds of a turn signal and a windshield wiper. Anywhere circular motions of different diameters coincide there is an opportunity to calculate their least common multiple and reconcile them through their lowest common denominator. Is it not useful to mimic these motions in our music?

[illegible]

Example 12. Note that the conductor and viola use identical fingerings. Students have suggested that substituting a melody for the finger numbers makes them easier to memorize. For example, the left-hand pattern is: do re mi fa sol fa mi re mi fa sol fa.

Conductor	1																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								</
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Example 13