Transformational Theory in the Undergraduate Curriculum: A Case for Teaching the Neo-Riemannian Approach[1]

Nora Engebretsen and Per F. Broman

In recent years, neo-Riemannian or tonal transformational theory has generated considerable interest within the academic music theory community, primarily due to insights it offers into the organization of passages that are triadic but not functionally coherent.[2] With the 2003 publication of Miguel Roig-Francolí's *Harmony in Context*, neo-Riemannian theory made its début in an undergraduate harmony textbook, appearing as part of a chapter devoted to nonfunctional pitch centricity in late-nineteenth- and early-twentieth-century music.[3] As brief as Roig-Francolí's presentation is, any inclusion of neo-Riemannian theory in an already crowded undergraduate curriculum raises the question: what can our students gain through study of the neo-Riemannian approach?

This paper provides an introduction to neo-Riemannian theory, outlining the material we choose to cover with our students, including analytical examples and composition and aural skills exercises departing from our experiences in the classroom, and presents arguments in support of incorporating the neo-Riemannian approach into the undergraduate core curriculum. We suggest that the study of neo-Riemannian theory serves at least two pedagogical ends: first, it serves well as a capstone to undergraduates' study of tonal harmony, insofar as it leads students explore notions of tonality and musical coherence and also to engage the limits of analytical systems designed to model functional tonal organization (understood here in a broad sense, as embracing normative harmonic and contrapuntal procedures);

[1] An earlier version of this paper was read at the College Music Society Annual Meeting, San Franscisco, on November 4, 2004. We are grateful for the comments we received on this occasion, as well as for those from the three anonymous readers on JMTP's Editorial Review Board.

[2] In addition to the works listed in our references, see the *Journal of Music Theory* 42/2 (1998), which was devoted entirely to this topic.

[3] Roig-Francolí 2003, 863–71. Roig-Francolí avoids the need to explain the historical association carried by the "neo-Riemannian" label by instead referring directly to the "PLR model" and "parsimonious voice leading."

and second, neo-Riemannian theory can help students bridge the gap between common-practice harmony and twentieth-century techniques through the introduction both of general notions about the basis of musical coherence and of specific concepts central to set theory and collectional approaches.

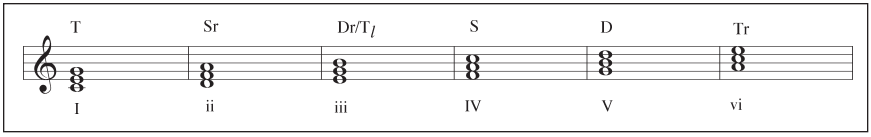
As our presentation will suggest, we favor introducing the neo-Riemannian perspective toward the end of our students' exploration of chromatic harmony, in the context of a survey of linear chromatic techniques, which is in keeping with Roig-Francolí's placement of the topic in his text, and then revisiting the neo-Riemannian approach, in conjunction with neo-tonal repertoire, as part of a survey of analytical approaches to twentieth-century music. This scheme encourages students to consider matters of tonal coherence in greater depth and from two different historical perspectives; however, we believe that many of the benefits discussed below accrue with even brief exposure in the context of an introductory chromatic harmony course.

Teaching the Neo-Riemannian Approach: Some Preliminaries

Richard Cohn (1998a, 169) has framed the central question motivating neo-Riemannian theory as follows: "if this music [music that is triadic but functionally indeterminate] is not fully coherent according to the principles of diatonic tonality, by what other principles might it cohere?" The neo-Riemannian response recasts elements of Hugo Riemann's late-nineteenth-century harmonic theories within a transformational framework to account for relationships among triads on the basis of voice leading rather than root progression or function. Although the neo-Riemannian transformations can be defined expediently without reference to Riemann's function theory—indeed, this is the course that Roig-Francolí chooses in his presentation—our preference is to engage the historical precedent. Our reasoning is twofold: first and foremost, we find that this approach encourages students to relate the neo-Riemannian transformations to familiar diatonic progressions, strengthening their sense of a connection between the functional progressions they have studied and the non-functional successions to be modeled using the neo-Riemannian techniques; and second, we find that such historical "digressions" pique the students' curiosity about different theoretical traditions.

Riemann's system of harmonic functions shares certain traits with Roman numeral analysis. Notably, both describe the relationship of the diatonic triads to their referential tonic, but whereas Roman-numeral labels reflect scalar ordering, Riemann's labels convey a hierarchical interpretation of primary and secondary chords. Riemann's system emphasizes three fundamental or primary triads—Tonic, Subdominant, and Dominant—from which all other chords derive. The remaining triads—the secondary triads—are understood to derive from one or more of these three fundamental triads via Relative, *Leittonwechsel*, or Parallel relationships.[4]

Example 1 gives the Riemannian labels for the diatonic major and minor triads in a major key. The triad on the sixth degree, for instance, is normally called the "Tonic Relative" (Tr). This is familiar enough to students from their knowledge of relative keys, and with prompting they can easily identify the triad on the second scale degree as the "Subdominant Relative" (Sr), reflecting what they already know to be a close connection between IV and ii(6). The labeling of the triad built on the third scale degree is a little more complicated. Following the same logic applied in labeling the supertonic and submediant triads, the mediant triad would be the "Dominant Relative" (Dr). This label is often problematic, however, in that it implies a close functional relationship between iii and V, even though the mediant triad is not necessarily affiliated with the dominant. (Riemann's theory puts a great stress on the way we perceive a particular chord to behave within a progression, so it is important to label the chords to reflect their functions.) The more common role of iii is as a substitute for or as an extension of I, as in the progression I–iii–V or I–iii–IV. For this reason, the mediant is often labeled in a way that emphasizes this connection to the Tonic.



Example 1

[4] The Relative, *Leittonwechsel* and Parallel relations are Riemann's *Parallel*, *Leittonwechsel*, and *Variante* relations, respectively. To avoid confusion, we use only the standard English names, here and in class. All three relationships are presented in Riemann's *Vereinfachte Harmonielehre* of 1893, but the Parallel relationship does not appear in Riemann's functional symbology until 1918, in the sixth edition of the *Handbuch der Harmonielehre* (Mooney 1996, 234–35).

The standard Riemannian term for this relation is *Leittonwechsel* or "leading-tone change"—the root of the major triad is displaced down by half step to yield the Tonic *Leittonwechsel* chord (T_l).[5] The final relationship that Riemann invokes, the Parallel, is again a familiar one from the study of key relationships, and students also immediately associate it with the simple modal alterations they have studied, such as the use of the minor subdominant in a major key or of the Picardy third in a minor key.

The relationships just described are part of Riemann's original harmonic theory. How does this all relate to *neo*-Riemannian theory? David Lewin initiated the neo-Riemannian project in his seminal 1982 article, "A Formal Theory of Generalized Tonal Functions," and further developed his ideas in his 1987 book *Generalized Musical Intervals and Transformations*. In these works Lewin introduced the notion of a transformational approach to triadic relations and also forged the connection between this approach and Riemann's theory. Brian Hyer and Richard Cohn have built upon Lewin's work. Cohn, in particular, has been instrumental in establishing Riemann's Relative, *Leittonwechsel*, and Parallel relationships as the fundamental transformations of the neo-Riemannian approach and in exploring the musical—and mathematical—potential of their combinations.[6]

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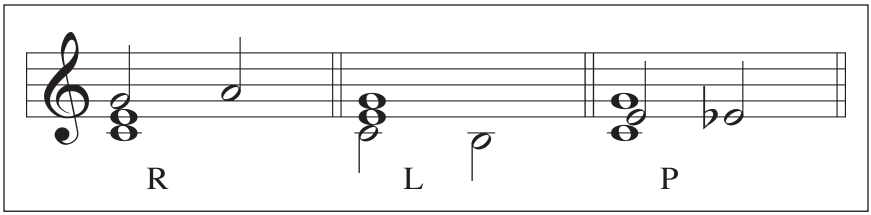
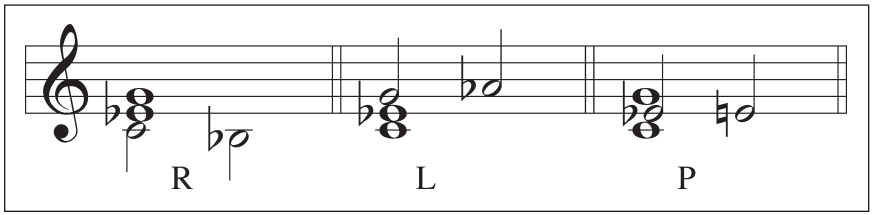
[5] Riemann indicates the Tonic *Leittonwechsel* chord by superposing the symbol "<" over the T. We prefer the subscripted "*l*" in our teaching, as its meaning is more directly apparent and also because it avoids the issue, at this stage, of the difference between major and minor *Leittonwechsel* chords. In a minor key, the tonic's *Leittonwechsel* chord would involve an upward half-step displacement of the tonic's fifth (e.g., the *Leittonwechsel* chord of an A-minor tonic would be an F-major tonic, with E being displaced by F) and would be indicated by superposition of the symbol ">" over T. The subscripted *l* avoids discussion of two different symbols for the *Leittonwechsel* relation.

[6] Riemann's Relative, *Leittonwechsel*, and Parallel relationships appear in Lewin's works (as REL, LT, and PAR) in conjunction with a number of other relationships, including DOM, SUBD, MED, SUBM, and SLIDE (Lewin 1987, 176–8). Hyer (1989, 1995) focuses on the interaction of REL, LT, PAR, DOM and SUBD/DOM^{-1}—which he re-labels R, L, P, D and D^{-1}—and explores the structure of the mathematical group that they generate. Cohn (1996, 1997) drops Hyer's D and D^{-1} to focus on the R, L, P and their compounds. For a more complete account of the development of neo-Riemannian theory and a critical comparison of various neo-Riemannian systems, see Kopp 2002, 142–64.



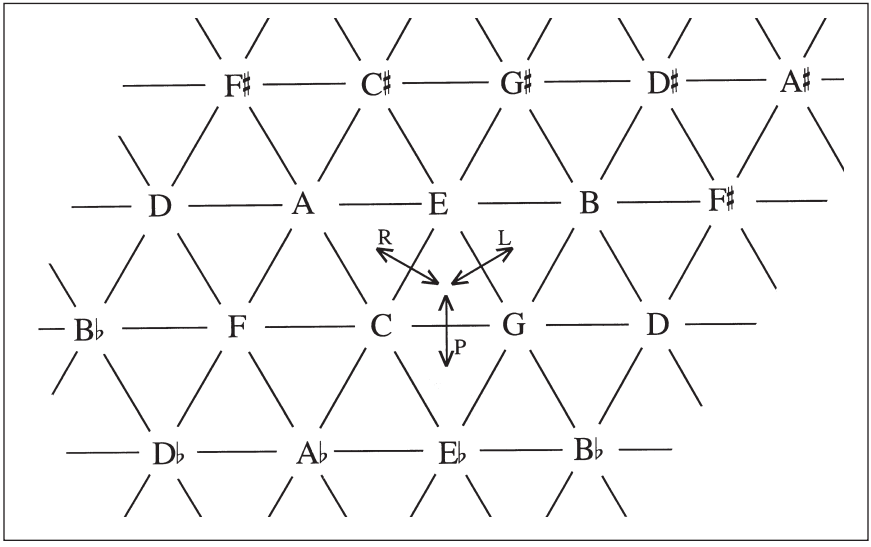
Example 2a



Example 2b

Neo-Riemannian theory appropriates Riemann's Relative, *Leittonwechsel*, and Parallel relationships—which are usually referred to in abbreviated form as R, L, and P—and recasts these relationships in dynamic terms as voice-leading transformations. These transformations have been formalized in the literature, but for our purposes, definition by example will suffice. As shown in Example 2a, the R operator transforms a C-major triad into an A-minor triad and vice versa; the L operator transforms a C-major triad into an E-minor triad and vice versa; and the P operator transforms a C-major triad into a C-minor triad and vice versa. All three operators are involutions—meaning each is its own inverse, that is, each undoes itself. Example 2b shows the results of applying the R, L, and P transformations to a C-minor triad, rather than to a C-major triad as in Example 2a. The examples highlight the different effects the transformations have when applied to major and minor triads, and specifically illustrate the vertically mirrored voice leading. The RLP transformations are usually characterized in terms of the parsimonious voice leading they entail—in each case, two notes are preserved while a single voice moves by step to effect the change of harmony. R preserves the triads' major third; L preserves the triads' minor third, and P preserves the triads' fifth. R involves whole-step motion in the remaining voice, whereas L and P involve half-step motion. This view of the transformations connects directly with the normative common-practice voice-leading procedures that the students have internalized through their study of part writing.

In addition to redefining the original Riemannian relationships as transformations, neo-Riemannian theory discards their reference to diatonic context: R transforms a C-major triad into an A-minor triad, whereas Riemann would have classified the A-minor triad with respect to a particular key, as the Tonic Relative in the key of C or as the Subdominant Relative in G, and so forth. This is important in that the abandonment of diatonic context allows neo-Riemannian theory to accommodate a full range of chromatic relationships among triads.



Example 3

The connection between any two given triads is easily identified through reference to the *Tonnetz* or Table of Tonal Relations, given in Example 3, which serves as a map of LPR relations. In more formal terms, the *Tonnetz* constitutes the geometry for the mathematic group associated with the action of the LPR transformations on the consonant triads—a group structurally analogous to the standard T_n/T_nI group of atonal set theory. On this table, which appeared in various forms in nineteenth- and early-twentieth-century harmony treatises, triads are represented as triangles and the LPR transformations can be visualized as flips across the triangles' edges.[7] The two notes preserved as common tones lie on the axis about which the triangle flips: L flips triads across the upper-left to

[7] On the history of the *Tonnetz*, see Mooney 1996.

lower-right axis, P flips triads across the horizontal axis, and R flips triads across the lower-left to upper-right axis. Nineteenth-century theorists generally regarded the *Tonnetz* as a map of relationships under just intonation, but neo-Riemannians almost exclusively use the equal-tempered version. Under equal temperament, each pitch class appears just once on the *Tonnetz*; the plane shown here should be understood to wrap around so that repeated or enharmonically equivalent note names occupy a single point on the surface of a torus—a donut shape. Thus, examining the *Tonnetz*, as we trace an RPRP (R-then P-then R-then P) path from the central C-major triad to the F♯-major triad northwest of it on the Table, it should be understood that the enharmonically equivalent B♭ in the lower right of the diagram supplies the A♯ that would complete the F♯-major triad. Neo-Riemannians refer to this sort of mapping of progressions on the *Tonnetz* as "navigating the *Tonnetz*."[8]

As an introductory exercise, students are encouraged to explore the *Tonnetz* by identifying the shortest path between a given pair of triads, as in the example above, by calculating the result of applying a given compound transformation (such as RPRP) to some given triad, and, as Roig-Francolí recommends, by composing short progressions based on various combinations of L, P, and R—on paper and/or improvising at the keyboard.

Another preliminary exercise we have found helpful in familiarizing students with the LPR operations, in reference to the Table or not, is an ear-training exercise. We divide the class into three sections, ask each section to sing a different member of a specified major or minor triad, and then give them a neo-Riemannian operator. The students have to realize which chord member they are singing (root, third, or fifth) and what to do when asked to change the chord via R, L, or P. Although the individual transformations are relatively simple to perform in isolation, when they are applied in succession, this exercise can prove quite difficult. The R and L operators pose particular difficulties in this context, as they affect different chord members depending upon the quality of the triad to which the transformation is applied. The level of difficulty also depends on the specific combination of transformations selected and the relative functionality of the resulting progression. Departing from a C-major triad, for example, the transformations R, then L, then P yield the succession C→Am→F→Fm, which is

[8] After Cohn 1997.

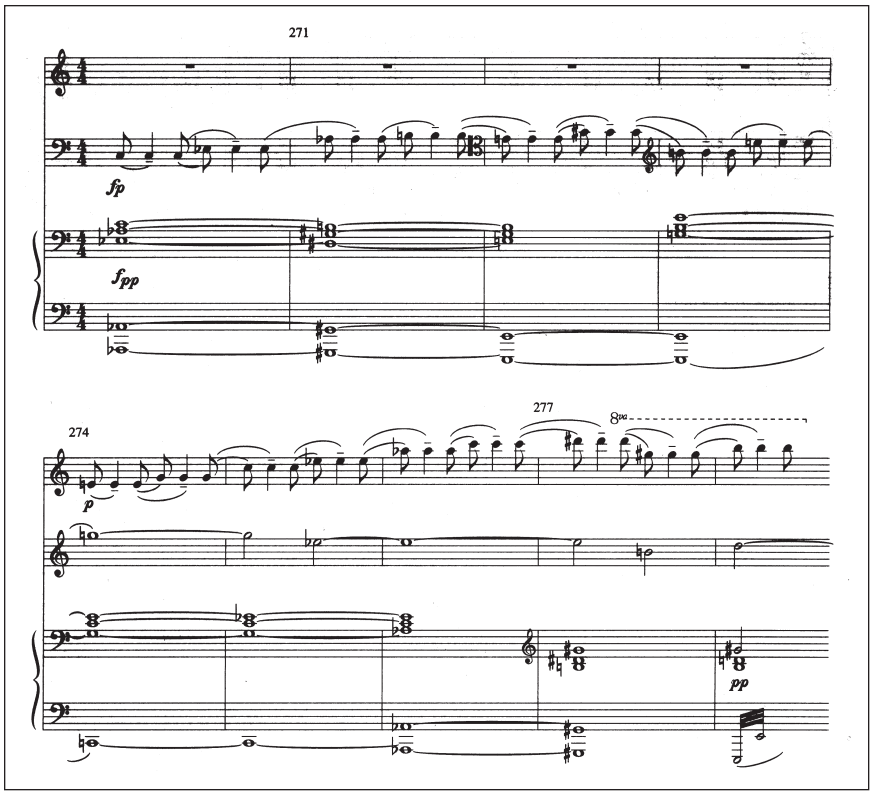
a familiar functional progression (I–vi–IV–iv) in C-major, whereas the transformations P, then R, then P, then L yield the succession C→Cm→E♭→E♭m→C♭, which is difficult to shoehorn into any meaningful functional context and is not a succession that students are likely to have encountered in their aural skills work.

Teaching the neo-Riemannian Approach: Generated Cycles and Non-Functional Successions

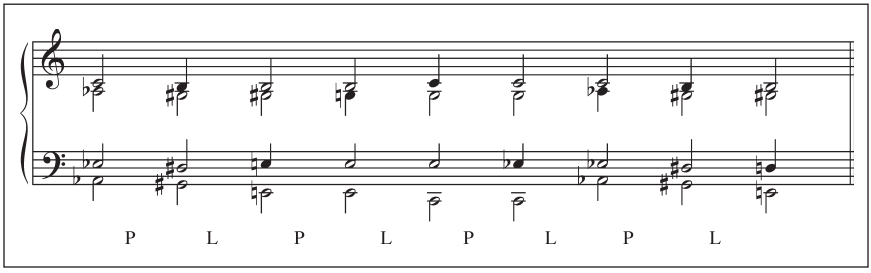
As students develop facility with the LPR transformations and *Tonnetz* navigation, they quickly discover that it is possible to move between any two of the twenty-four major and minor triads in five steps or less. The possibility of connecting even the most remotely related chords recalls the criticism leveled by the nineteenth-century theorist Arthur von Oettingen that this kind of approach leads to a "chaos of possibilities" (1866, 156). Guided by musical practice, however, neo-Riemannian analysts have tended to privilege patterned motion that involves parsimonious voice leading at each step and results in cyclic closure. Cohn, in particular, has been instrumental in defining an analytical practice centered on combinations of the LPR transformations that yield closed cycles of triads.[9] Three such cycles are generated via pair-wise applications of LPR transformations—via "binary generators" in Cohn's (1997) terms—and two of these binary cycles constitute more tractable subgroups of the full LPR group. A fourth cycle type, generated by the repeated application of all three transformations in some fixed order, does not represent a subgroup but is nevertheless very much of interest musically.

Much of Cohn's analytical work has focused on what he describes as "maximally-smooth" cycles generated by alternately applying L and P—the two transformations involving half-step displacements. In "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," Cohn offers an example of a complete LP cycle found just prior to the recapitulation in the first movement of Brahms's Double Concerto (mm. 270–77). This example, which also appears in Roig-Francolí's text, is reproduced as Example 4: Example 4a gives the score and 4b a reduction showing the essential voice leading underlying the progression (the single pitch that changes from one chord to the next is represented with a solid note head).

[9] Our presentation, like Roig-Francolí's, follows from the discussions of these cycles in Cohn 1996 and 1997, in particular.



Example 4a



Example 4b

Before diving into a transformational reading—indeed, before even introducing the LPR approach—we ask our students to listen to and discuss the passage. Drawing on their recent studies in chromatic harmony, they are quick to note that the bass descends by major thirds, symmetrically dividing the A♭/G♯ octave, and that this symmetrical division entails motion through an enharmonic seam and is problematic from the perspective of diatonically

oriented Roman-numeral analysis. The concept of modal mixture is also readily invoked to describe the major/minor pairings of triads, though the initial A♭-major/G♯-minor pair does cause some confusion in this regard. With some prompting, students note that although the passage functions much like a sequence—prolonging the A♭ tonicized in m. 270 via a series of non-functional successions—it is not strictly sequential, and also that the most characteristic feature of the sustained accompaniment, other than the bass-line descent by thirds, is the parsimonious voice-leading motion between triads.

Once familiar with the LPR transformations, students readily focus on these voice-leading connections and describe the passage as comprising a cycle of triads alternately related by P and L transformations. As shown in Example 5, there are four such cycles, which partition the twenty-four major and minor triads into four discrete tonal regions.[10] Mapped on the *Tonnetz*, each LP cycle runs along a lower-left to upper-right alley. These cycles are often referred to as a hexatonic cycles because each engages all and only those pitch-classes forming a hexatonic collection. The pitch class content of each cycle is shown to its right on Example 5.[11]

P	L	P	L	P	L		
C	Cm	A♭	G♯m	E	Em	(C)	[3478E0]
D♭	C♯m	A	Am	F	Fm	(D♭)	[014589]
D	Dm	B♭	B♭m	F♯	F♯m	(D)	[12569T]
E♭	E♭m	B	Bm	G	Gm	(E♭)	[2367TE]

Example 5

[10] Cohn (1996) develops this characterization of the cycles as tonal regions, folding the four hexatonic cycles into a hyper-hexatonic system and then describing motion between cycles as modulatory. We do not include this material in our introductory presentations of the neo-Riemannian approach, nor does Roig-Francolí. Those interested in pursuing this topic with their students might also consult the analysis of part of the first movement of Schubert's Piano Sonata in B♭ given in Cohn 1999, in which Cohn aligns three of the hexatonic cycles with tonic, subdominant, and dominant functions.

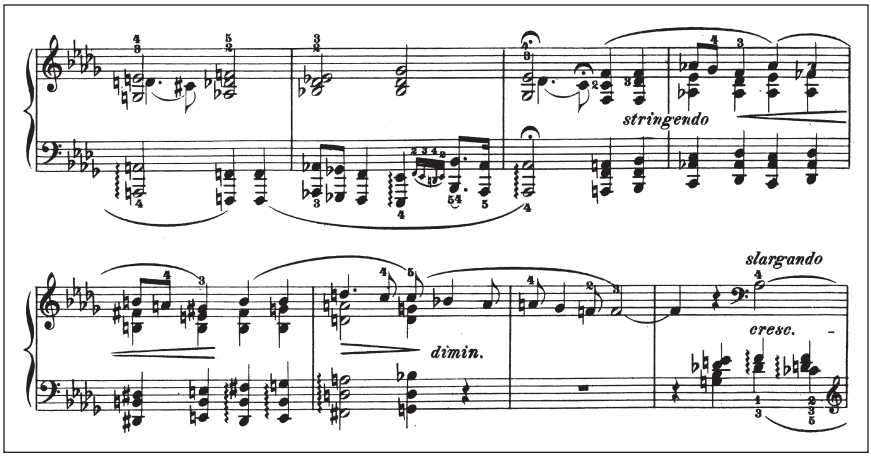
[11] Example 5 gives each cycle's pitch-class content in pc numbers; in class we use letter-names, as we do not introduce pc numbers until the final semester of our theory core.

We next ask our students to determine if similar cycles are generated when either L or P alternates with the slightly less parsimonious R transformation, which involves a whole-step rather than half-step voice-leading displacement. Applied in alternation, the P and R transformations partition the consonant triads into three cycles of eight triads each, as shown in Example 6. Whereas each LP cycle exhaustively engages a unique hexatonic collection, each PR cycle exhaustively engages a unique octatonic collection. Likewise, whereas LP cycles feature an overall pattern of root motion by major third, PR cycles—which thread along the upper-left to lower-right alleys of the *Tonnetz*—feature overall root motion by minor third. Cohn (1997, 35) offers an example traversing a complete PR cycle taken from the opening Andante of the Overture to Schubert's opera *Die Zauberharfe* (also known as the "Rosamunde Overture"), mm. 8–32.

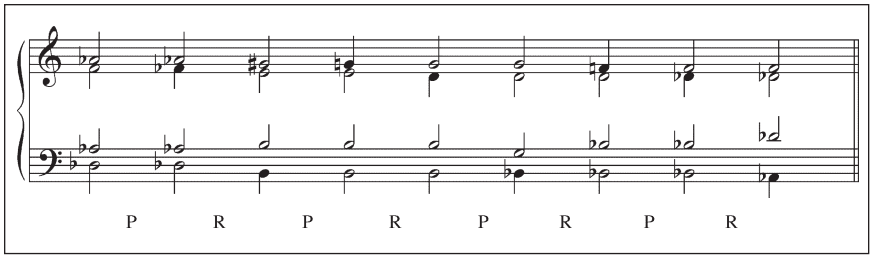
P	R	P	R	P	R	P	R		
C	Cm	E♭	E♭m	F♯	F♯m	A	Am	(C)	[0134679T]
D	Dm	F	Fm	A♭	G♯m	B	Bm	(D)	[235689E0]
D♭	C♯m	E	Em	G	Gm	B♭	B♭m	(D♭)	[124578TE]

Example 6

The LR cycle, which moves along the horizontals of the *Tonnetz*, differs from the LP and PR cycles in that it progresses through all twenty-four consonant triads, rather than partitioning them into shorter cycles. Cohn (1997, 36) characterizes the full cycle as being "too long to sustain compositional interest," and indeed the LR cycle is usually presented in a significantly truncated form. Cohn and Roig-Francolí nevertheless illustrate the cycle with an exceptionally long, 19-chord presentation drawn from the second movement of Beethoven's Ninth Symphony (mm. 142–71).



Example 7a



Example 7b

In our initial consideration of the three binary-generated cycles, we choose to present straightforward, cyclic examples in which successions modeled by the L, P, and R transformations occur as foreground events, highlighting the parsimonious voice-leading connections between chords as the sources of the successions' coherence. The neo-Riemannian approach need not be applied so restrictively, however. Roig-Francolí illustrates the PR cycle with a slightly more complicated example, from Liszt's *Consolation*, no. 4. Example 7 reproduces the relevant portion of the score (Example 7a) and Roig-Francolí's voice-leading reduction of the full cycle around which the passage is organized (Example 7b). Two aspects of this example are particularly noteworthy. First, the PR cycle here does not appear on the absolute foreground, but instead comprises the harmonic goals of a series of tonicizations (each member of the cycle is preceded by its own dominant, a technique familiar to students from their study of chromatic sequences). Second, the passage includes what Cohn (1996, 21) terms "mixed" motion—that is, not strictly cyclic motion—through the PR system. The mixed

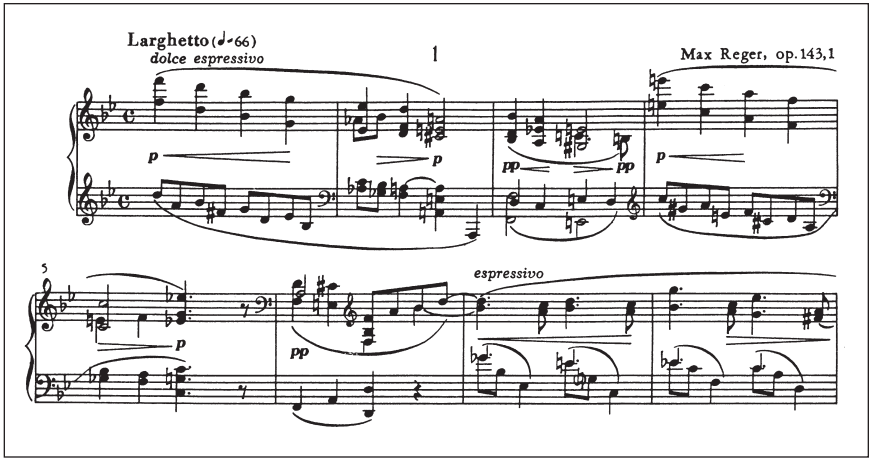
motion here involves a skip over one step in the cycle (the G-major triad) and then a skip over two more steps on the way back to the opening D♭-major triad. (The familiar practice of skipping a segment of the diatonic ascending fifths sequence can provide students with a point of reference for these sorts of skips with respect to an underlying pattern.) We follow in-class discussion of this passage with an examination of Schubert's "Der Jüngling und der Tod" (D. 545), which features similar, relatively straightforward, mixed motion through a PR cycle but at a deeper middleground level.[12]

Mixed motion through cycles and the participation of cycles in middleground structure both provide openings for discussions about the notions of tonal organization implicit in the neo-Riemannian approach, as well as those associated with more functionally oriented linear/harmonic models. For instance, the coherence of complete, foreground cycles can be understood through reference to smooth voice leading, realized on the musical surface (as in Example 4), but can these voice-leading connections continue to serve as a source of coherence in a passage involving mixed motion, in which the underlying voice-leading cycle may or may not be easily intuited, depending on the order in which the triads are presented? How important is cyclic closure in creating a sense of coherence? How is tonal distance gauged in cyclically based contexts versus in functional contexts? The presence of middleground cycles, as in Example 7 and particularly at deeper levels, invites discussion of the intelligibility of embellished cycles and of interactions of different systems of tonal organization on different structural levels, and perhaps (if appropriate in the context of the course) comparison to traditional Schenkerian models of tonal organization.[13]

[12] Our discussion of this song follows the analysis presented in Siciliano 2005b. For those who prefer not to introduce the ideas of middleground cycles, mixed motion, and PR cycles all at the same time, Cohn 1996 provides examples of mixed motion through foreground LP cycles, including that of the more extensive mixed motion found in The Grail motive from *Parsifal*, a passage frequently analyzed in the neo-Riemannian literature. Siciliano 2005b presents an example of a complete, shallow-middleground LP cycle drawn from Schubert's E♭-major Trio (D. 929), which would also provide a nice transition from a simple LP cycle, as shown in Example 4, and the PR cycle shown in Example 7.

[13] Felix Salzer's (1962) extensions of Schenker's ideas to include "completely contrapuntal structures" and a range of prolongational practices, as well as his notion that "[t]onality is synonymous with chord prolongation" (p. 232), might also be invoked in this context.

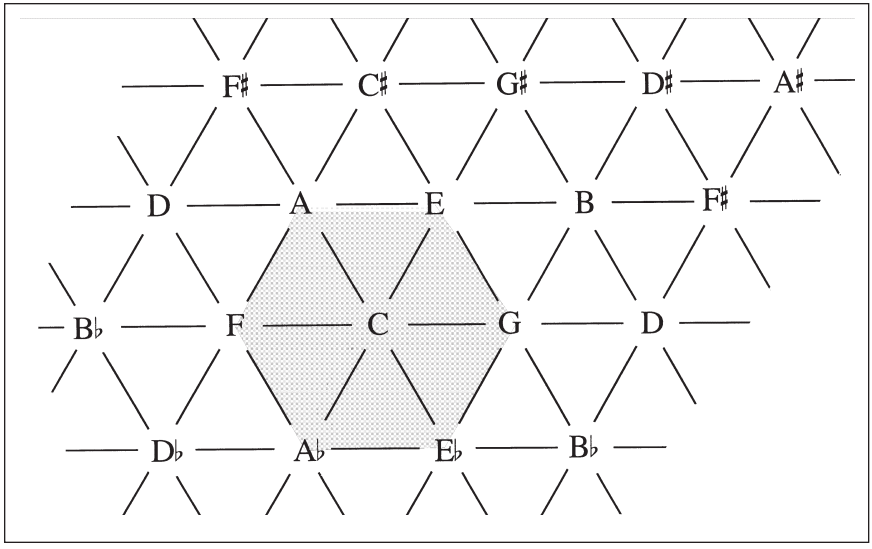
As noted above, both LP cycles and PR cycles engage symmetrical collections—the hexatonic and octatonic collections, respectively. As we examine passages organized around these cycles in class, we take the opportunity to return to previous discussions of tensions between the asymmetries of the diatonic collection and symmetrical divisions of the octave, and again to stress the importance of considering how these cyclically based passages are integrated into predominantly functional frameworks and into the formal structures of the pieces in which they appear. In Example 4, for instance, the cycle sets up the arrival in m. 278 of the retransitional dominant by prolonging A♭, the enharmonic equivalent of the leading-tone in the home key of A minor, which is then incorporated into the dominant-seventh on E.



Example 8

As we discuss the LR cycle, which does not symmetrically divide the octave, our primary focus is on the relationship of the cycle to familiar, functional harmonic routines, underscoring the continuities between the LPR model and earlier studies of diatonic harmony. An example we find useful in this regard is the first piece from Reger's *Träume am Kamin*, which is given in Example 8. The piece begins with a portion of an LR cycle: D minor, B♭ major, G minor, E♭ major, with applied leading tones on off beats separating the chords. From E♭ major, we expect the cycle to continue on to C minor, A♭ major, and so on. The expected C-minor triad is omitted, and the next chord we hear is A♭ major. The initial portion of the cycle could be interpreted as a progression from the D-minor tonic

through a series of pre-dominant chords (VI–iv–♭II), reflecting the largely diatonic organization of short segments of the LR cycle, but it is not clear how the A♭-major triad could be accounted for without reference to the cycle. The return of the D-minor tonic in bar 2, in place of the expected F-minor triad, interrupts the cycle which gives way to a half cadence on A, bringing the opening gesture to a close. The initial theme returns in measure 4 supported by an LR cycle, now in A-minor: A-minor—F-major—D-minor. The expected B♭-major triad is omitted, but rather than skipping ahead to the next member of the cycle, G-minor, as the original statement did, the cycle is abandoned here. While the harmonization of the introductory gesture still follows the LR pattern, it now uses a short enough segment of the cycle—just three chords—that reference to the cycle is not necessary: the progression can be accounted for in terms of normal diatonic practice. This opening gesture appears several times during the course of the piece, with and without the cyclic harmonization.[14]



Example 9

Though coverage of the individual LPR transformations, their *Tonnetz* representations, and binary-generated cycles certainly would provide a sufficient introduction to the neo-Riemannian

[14] Roig-Francolí includes an analytical exercise in his workbook (Example 30.1, Chopin's Ballade no. 1 in Gm, mm. 90–95) that uses an LR cycle fragment in a less functional setting.

approach, we choose to include examples of ternary-generated cycles in our teaching as well, due to their musical interest and also to lay a foundation for a return to neo-Riemannian techniques in conjunction with neo-tonal repertoire during the final semester of our theory core.[15] These cycles, generically dubbed "LPR loops" by Cohn (1997, 43), are generated by the repeated application of all three of the LRP transformations in some fixed order, and they consist of all and only those six triads sharing a single pitch class in common. When mapped on the *Tonnetz*, these cycles form a loop around that single shared pitch class—Example 9 shows the LPR loop around C, which comprises the triads C, Cm, A♭, Fm, F, and Am.[16] Cohn notes that these cycles provide ideal, parsimonious harmonic support for a sustained pitch, and he cites several examples from nineteenth-century operatic repertoire, adding "Such progressions with their implications of inner action or turmoil beneath a placid and harmonious surface, were well suited to symbolize nineteenth-century notions about the relationship of the inner and outer worlds."[17]

Why Teach the Neo-Riemannian Approach?

The introduction to neo-Riemannian theory outlined above requires roughly two weeks to work through—admittedly a substantial amount of class time in a semester already typically

[15] Roig-Francolí does not directly address LPR loops in his presentation of the LPR model, but encourages their discussion by including a passage featuring a fragment of LPR cycle as an analytical exercise in his workbook (Example 30.2, Verdi's, "Ah! Sì, ben mio," from *Il trovatore*, Act III, mm. 1–22).

[16] The specific order of the L, P, and R transformations depends upon the cycle's starting point and the direction of motion around the loop. The ternary generator PLR would produce the cycle shown here, departing from the C-major triad.

[17] Cohn 1997, 44–45. In addition to Verdi's "Ah! Sì, ben mio" from *Il trovatore* (see footnote 15), Cohn also examines Wagner's *Engelmotiv* from Act III of *Parsifal*, which features mixed motion through an LPR loop about A♭/G♯, and cites the opening of the Monks' Chorus from Verdi's *Don Carlos*, which features a fragment of the LPR loop about F♯/G♭. He also mentions the opening of Liszt's "Il Penseroso" (*Années de Pèlerinage, Deuxième Année*), which features mixed motion among four members of the LPR loop about E. Siciliano 2005b presents an LPR-based analysis of Schubert's "Trost" (D. 523). Those interested in including popular music in their classes can find an example of an LPR loop, from the chorus of "Shake the Disease" by Depeche Mode, in Capuzzo 2004, example 2.

packed with a survey of various chromatic chords and linear processes, modulatory techniques, and stereotypical formal patterns. Indeed, one might reasonably question whether an approach designed to address a rather narrowly circumscribed repertoire—tonally indeterminate but predominantly triadic passages—is worth this time investment, especially as we have seen that aspects of some such passages might be addressed through reference to familiar topics such as equal divisions of the octave. Our answer is resoundingly "yes," and we return to our initial question: what can our students gain through study of the neo-Riemannian approach?

In the context of a semester devoted to the study of chromatic harmony, neo-Riemannian theory can serve a synthetic function, insofar as it draws upon and integrates into a coherent system aspects of a number of topics often presented in isolation—topics including modal mixture, equal divisions of the octave, chromatic sequences, and common-tone retention and semitonal voice leading as general principles. The resulting system is by no means applicable to all chromatic repertoire—indeed, the familiar topics it engages have broader applications taken individually—but it does offer insights into the organization of select passages that might otherwise be explained vaguely as "coloristic chord successions."[18] Moreover, it encourages students to move beyond an atomistic focus on certain chord types and procedures, deployed within a predominantly functional framework, and to embrace the possibility of another kind of tonal organization.

This realization that a break with traditional, functional tonal organization need not lead to chaos, but might instead make way for coherent organization according to some other set of principles, is a realization we hope our students will come to as we lead them through a survey of twentieth-century techniques. Typically, however, we ask them to make this leap at the same time they are grappling with and often overwhelmed by the nomenclature and

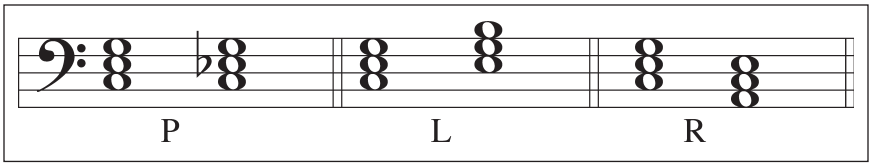
[18] Kostka and Payne 2004, 43ff. They recommend the default analytical technique of labeling root and sonority type. Clendinning & Marvin (2004, 606) are somewhat more specific about the basis of organization of these passages, which they discuss under the rubric of "Linear chromaticism" (chapter 29, Chromaticism): "In the Romantic era, some composers took the voice-leading idea even further, writing pieces where long spans of music consist of linear chords held together by their smooth, chromatic voice leading without much, if any, sense of progression or root motion."

concepts of set theory and the associated shift from a chordal/contrapuntal focus to a collectional focus (although there is no inherent contradiction between the two). Herein lies what we see to be the greatest benefit of teaching the neo-Riemannian approach: neo-Riemannian theory provides an intuitive way to bridge gaps between tonal and post-tonal techniques by laying the groundwork both for the acceptance of a model of coherence associated with transformations and their group-theoretic structure, and also for a gentle introduction to concepts such as inversion and set-class through reference to the familiar consonant triads.

Exposure to the neo-Riemannian approach in the context of chromatic harmony classes gives students an opportunity to explore a transformational perspective—to understand the P, L, and R transformations as "something one does to one triad to get another"[19]—in conjunction with familiar voice-leading routines between familiar chords, rather than in conjunction with the more abstract notions of pitch-class transposition and inversion, and under the additional burden of working with the less-familiar concept of pitch-class sets. Likewise, students' experiments with combinations of the LPR transformations, their confrontation of the "chaos of possibilities" presented, and their eventual understanding of some ways in which composers have contained this chaos, provide them with a frame of reference for processing the even greater chaos of possibilities posed by T_n/T_nI relationship among pitch-class sets. Moreover, the potential of the neo-Riemannian approach to supplement rather than supplant more traditional approaches to tonal music promotes the notion that multiple analytic approaches can co-exist and complement one another—a vital point in much analysis of twentieth-century music.

As important and potentially powerful as the connections between neo-Riemannian theory and set theory just described are, we do not find it necessary to dwell upon them in our teaching—we note the parallels and move on. There are, however, twentieth-century topics in relation to which we do make more direct and extended comparisons with neo-Riemannian theory. Among these are the concepts of pitch-class inversion and inversional equivalence, and the topic of symmetrical collections.

[19] This conception of transformation here follows Lewin's (1987, 177) explanation of a transformation as "something one does to a *Klang*, to obtain another *Klang*."



Example 10

Neo-Riemannian theory provides a nice introduction to inversion and inversional equivalence as the PLR transformations can be conceived of both as voice-leading transformations and as contextual inversion operations. As shown in Example 10, P inverts a triad so that its root and fifth map onto one another; L inverts a triad about its preserved minor third, flipping the major third from one side of the minor third to the other; and R inverts a triad about its preserved major third, flipping its minor third from one side of the major third to the other. These inversions are easily conceptualized in pitch space, through reference to staff notation, and in pitch-class space, through reference to the mod-12 clock face. As we map the results of PLR transformations on the mod-12 clock, we introduce the I^{x}_{y}-type labels as alternatives to the tonally oriented PLR labels. Once students understand the concept of inversion on the local, contextual level, it is a relatively small leap to the notion of inversion about an external referent, such as the 0–6 axis of the pitch-class clock face, and $T_{n}I$-type labels.

Insofar as inversional equivalence is concerned, we find reference to major and minor triads as objects of transformations to be useful both in highlighting the basis of the relationship and in conveying the loss of information that accompanies the shift from T_{n}- to $T_{n}I$-equivalence. The familiar description of major and minor triads' structure in terms of a minor third over a major third or vice versa—emphasized through the conception of PRL transformations as contextual inversions—helps to convey the notion of inversional equivalence, while students' hesitance to accept major and minor triads as being in some sense "the same" helps to sharpen the conceptual distinction between equivalence and identity.

The neo-Riemannian approach also provides an introduction to the symmetrical hexatonic and octatonic collections and their potential to support non-centric pitch organization. The introduction of these collections in the context of tonal harmony, rather than in the context of twentieth-century techniques, is particularly effective in that it encourages direct comparison of the symmetrical collections and the asymmetrical diatonic scale, and of associated

notions of tonality and tonal center—does cyclic closure, the return to start in a full, ordered presentation of an LP cycle, for instance, convey a privileged, even "tonic" status to that starting and ending chord? How does one know where one is, that is position find, in the cycles without any rare intervals? Even brief discussion lays a valuable foundation for study of these collections and centrality in twentieth-century contexts.

Revisiting the Neo-Riemannian Approach: Neo-Riemannian Transformations and Neo-Tonal Analysis

Beyond its utility in introducing twentieth-century analytic techniques, neo-Riemannian theory can itself be co-opted in the analysis of post-tonal repertoire. Some of these adaptations move beyond the scope of a typical undergraduate survey of post-tonal analytic approaches,[20] but we have found a return to neo-Riemannian techniques to be quite productive in conjunction with neo-tonal repertoire from the later twentieth century—a notion explored briefly in the most recent (2005) edition of Joseph N. Straus's *Introduction to Post-Tonal Theory*.[21]

Neo-tonal works, particularly those sometimes referred to as "neo-Romantic," often include passages that are triadic but tonally indeterminate—passages that in this respect resemble those Romantic-era passages around which the neo-Riemannian approach developed. Like their Romantic-era counterparts, these passages may include certain functional elements, such as anchoring cadences, but whereas the Romantic-era passages tend to be organized so as to integrate smoothly into otherwise fully functional frameworks, neo-tonal passages often appear in non-functional contexts or in some way mark their distance from the historical model. With this difference in mind, we ask our students to explore the organization of select neo-tonal passages, using neo-

[20] Michael Siciliano (2005a), for instance, re-conceives LP cycles in terms of the characteristic "toggling," rather than parsimony, of their voice-leading patterns, notes that toggling cycles of 3-3(014), 3-4(015), and 3-11(037) trichords all generate the hyper-hexatonic, and explores the applicability of his toggling-trichord approach to passages from Schoenberg's atonal works.

[21] We do not happen to use Straus's text in our undergraduate core, but neo-Riemannian theory's transformational perspective and focus on voice leading would, of course, connect very naturally with Straus's overall approach.

Riemannian techniques, and to compare and contrast their findings with the kinds of non-functional tonal organization encountered in Romantic music.

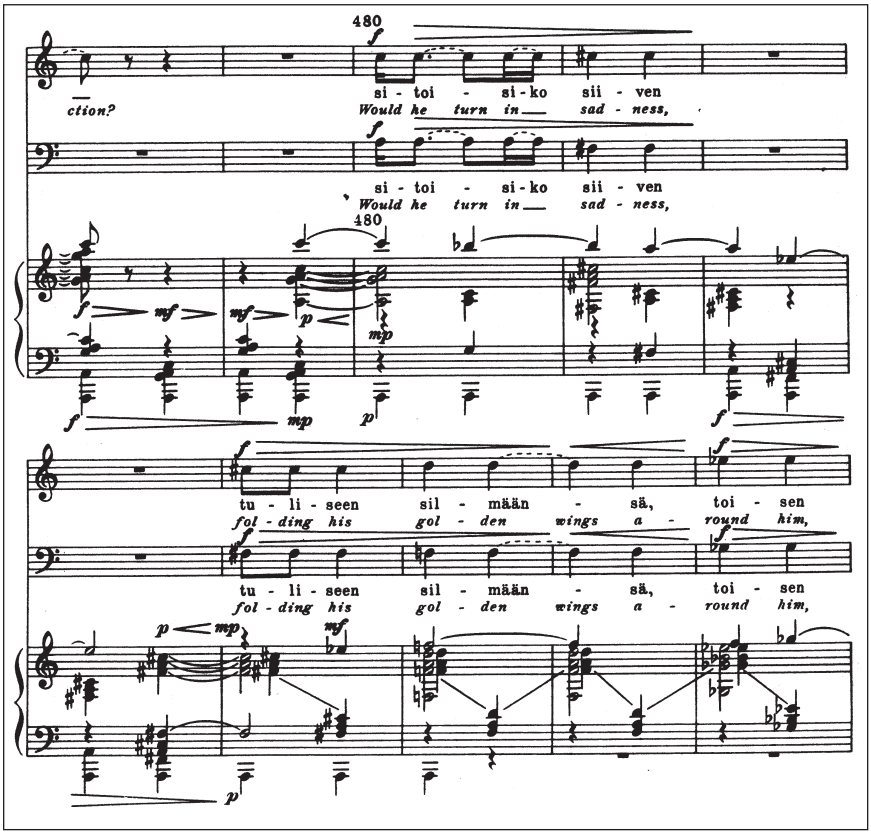
Although neo-tonal composers generally do avoid thoroughgoing use of functional tonal syntax, the disruption of functional tonality is not, given their historical position, their most pressing compositional concern. Instead, they grapple with the creation of their own alternative tonal languages and discovery of their own solutions to the challenges of projecting pitch centers and larger-scale tonal structures in the absence of a normative tonal practice. As such, the symmetrical hextatonic and octatonic cycles' destabilizing potential and capacity to support non-centric organization, which contributed to the rise of a fully chromatic tonal practice during the nineteenth century, hold less sway over neo-tonal composers.[22] Rather, the desire to project tonal centers and to sustain larger-scale structures leads neo-tonal composers down a somewhat different path, but one that also has precedents in nineteenth-century practices well modeled by the neo-Riemannian approach.

While a range of distinct tonal practices exist under the banner of neo-tonality, many neo-tonal composers seem to be attracted to tonal schemes well modeled by LPR loops. Neo-tonal composers' interest in these LPR cycles lies less in their potential for extra-musical associations or even for parsimonious voice leading—indeed, most neo-tonal composers seem not to privilege the maximally parsimonious LPR relations—and much more in their potential for projecting local tonal centers which in turn contribute in some way to larger-scale tonal organization. Here we present brief analyses of two passages reflecting two different uses of LPR loops that we use to spark discussion in class.

Finnish composer Aulis Sallinen's 1978 *Dies Irae* (for soprano, bass, male choir and orchestra) includes several predominantly triadic passages, including one that features rather straightforward use of LPR loops and also clearly illustrates one particular kind of role that LPR loops and their common-tone projections can play in non-functional tonal contexts. Example 11 (next page) reproduces mm. 478–501. As is often the case in neo-tonal works, the texture in the excerpt is layered, in the sense that the moving line at the

[22] Nevertheless, it is certainly possible to find hexatonic (LP) and octatonic (PR) cycles in this repertoire. Straus (2005), for instance, provides examples of passages based on LP cycles. See his Examples 4-27 through 4-30 and related discussions.

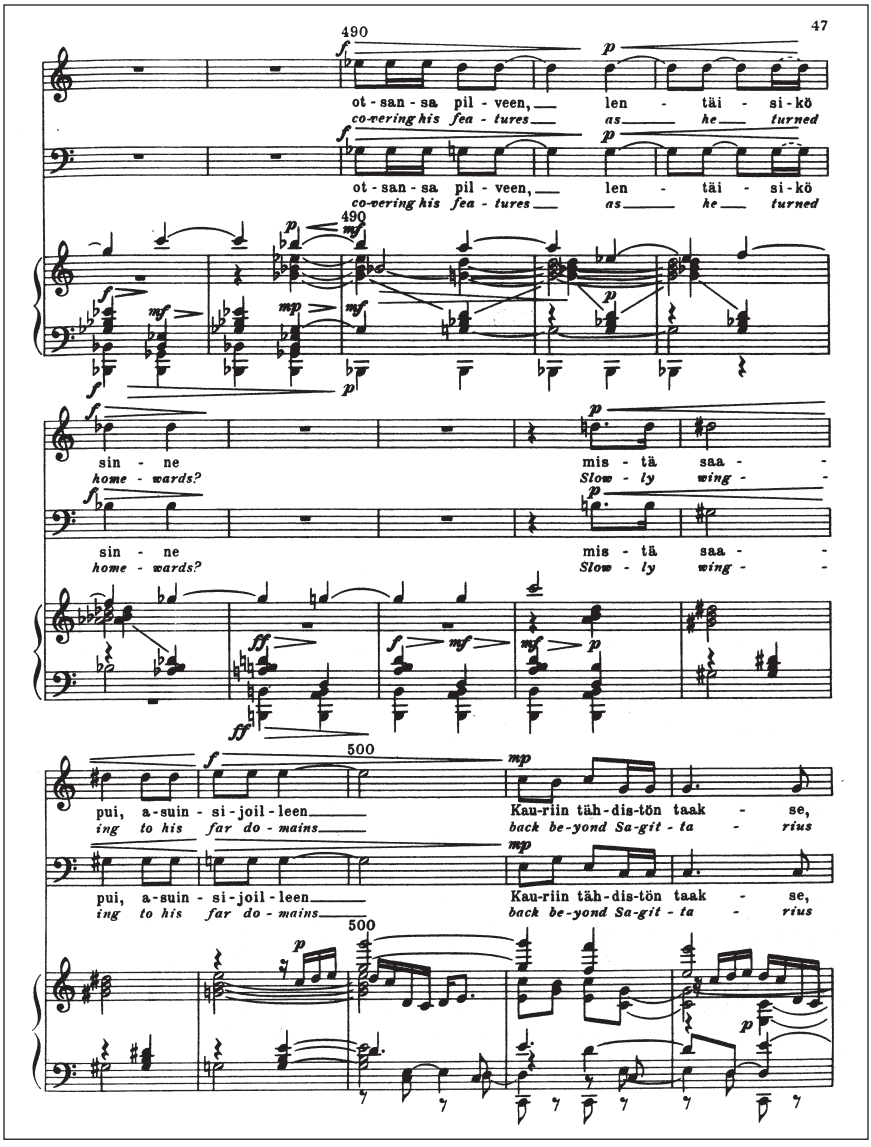
top of the orchestral reduction—beginning C–B♭–A in m. 479 onward—sounds against but is not included in the triadic analysis. The opening of this passage features a series of third-related minor triads (Am, F♯m, Dm) over a prominent bass pedal on the pitch A.[23] The three triads are all members of the LPR loop about A: Am–PR→F♯m–LP→Dm. Each triad is repeated in various registers before the next chord enters, but at the points where harmony changes, the voice-leading connections between triads are realized as parsimoniously as possible.



Example 11

The next 3 triads to enter, E♭m, Gm, and B♭m are members of the LPR loop projecting B♭, which enters as a pedal tone underneath

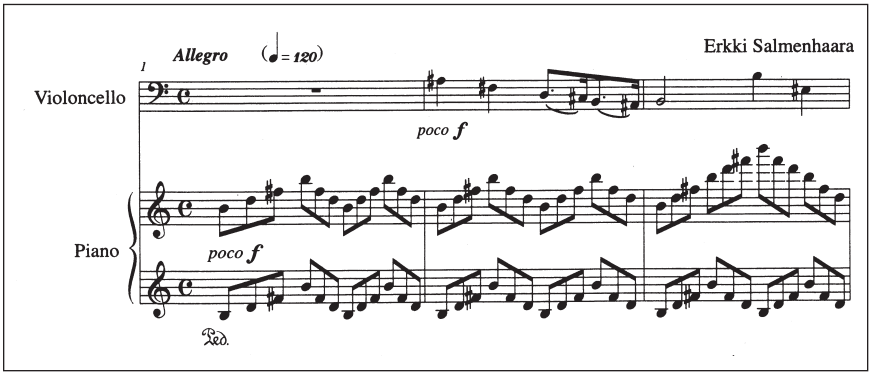
[23] The initial chord is actually an Am7th chord. Neo-Riemannian analyses often omit chordal 7ths, added 6ths and the like.



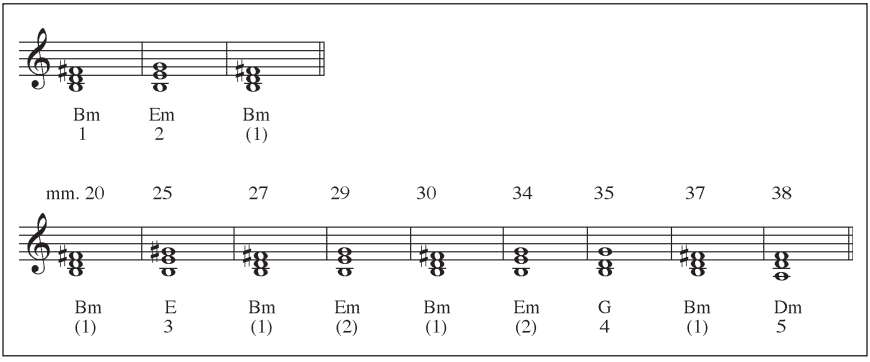
Example 11 *(continued)*

the progression in m. 488. This B♭ pedal eventually gives way to a pedal B♮, which is harmonized by the succession Bm, G♯m, Em—all members of the LPR loop about B. The chromatic ascent from one pedal tone to the next, A–B♭–B, projects a motive introduced at the outset of the piece, with the repetition of the chromatic ascent D♯–E–F over a B pedal.

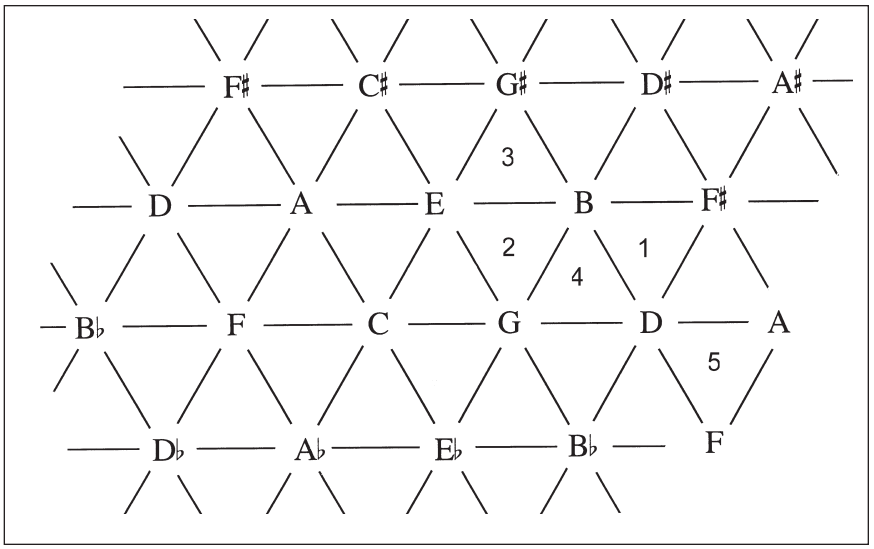
Finnish composer Erkki Salmenhaara also frequently engages LPR loops, often using the pitch-classes highlighted as common tones to create de facto prolongations and also to connect neo-tonal passages with tonal cadences, much as nineteenth-century composers would anchor their cycle-based progressions with strong cadences. The first movement of Salmenhaara's Sonata No. 2 for cello and piano (1982) presents what is initially a slowly unfolding triadic progression in near-constant arpeggiation against which melodic material is layered. Example 12 reproduces the opening measures to show the texture. As shown in the voice-leading reduction of mm. 1–38, Example 13a, the harmonic progression begins innocuously enough, with a plagal succession in B-minor, but as it grows increasingly chromatic it soon becomes difficult to interpret functionally.



Example 12

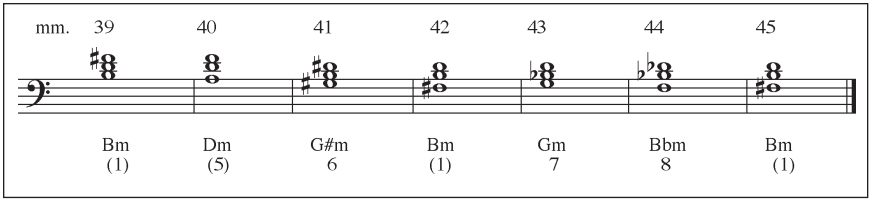


Example 13a

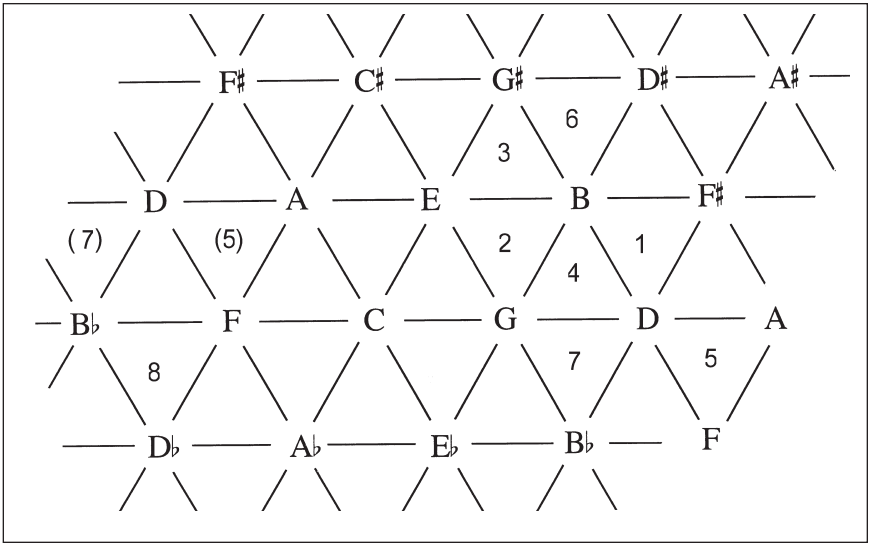


Example 13b

To facilitate mapping of the progression on the *Tonnetz*, chords have been numbered in Example 13, based on their order of initial entry, but note that these numbers do not always reflect order of succession: for example, after the second chord (E-minor) is heard in mm. 14–15 the opening B-minor triad returns in mm. 16–24 and it is from this B-minor triad that the next new chord, the E-minor triad in m. 25 is approached. Mapping the progression on the *Tonnetz*, Example 13b, reveals that the opening 38 measures consist of mixed motion—relatively smooth but generally non-parsimonious motion—within the LPR loop around the tonic B, which is temporarily abandoned upon the introduction of the D-minor triad in m. 38 (chord #5). When combined with the B-minor and G-major harmonies that immediately preceded it (#1 & 4), the appearance of the D-minor triad suggests a shift to the LPR loop around D, which interlocks with the original loop about B.



Example 14a



Example 14b

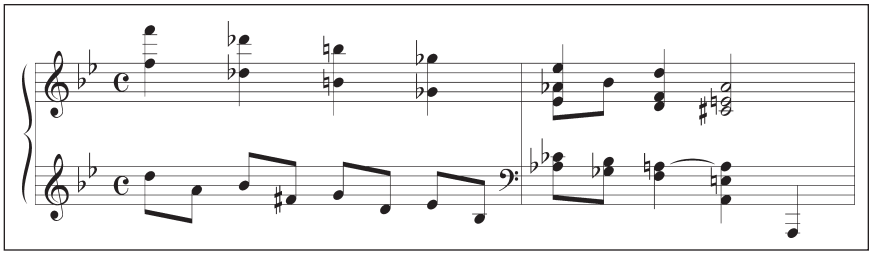
The next two new harmonies, shown in the continuation of the voice-leading reduction in Example 14a, the G♯-minor triad in m. 41 and the G-minor triad in m. 43 (#6 &7, respectively), each fit into one of the 2 interlocked cycles, Example 14b, but the move from Gm to B♭m in m. 44 (#7 to 8) suggests a shift into a third LPR loop, this time around B♭. This brief emphasis on B♭ is immediately countered by the return of the B-minor triad in m. 45, and in the next few measures, there is a clear return to the familiar territory of the LPR loop around D, as the piano arpeggiates the B-minor triad (#1) against which the cello arpeggiates first a D-minor triad (#5) and then a G-major chord (#4). This G-major chord is in fact a Mm7th chord, including an F♮ (the piano drops its F♯ at this point), which then resolves functionally to a C-major triad in m. 58. Thus, the pitch-classes emphasized within the two most prominent LPR loops—B and D—are eventually subsumed within a dominant-seventh chord on G, and in a sense point forward toward a clear,

functionally articulated harmonic goal, even if not all of the chord-to-chord successions in the passage are functionally coherent. (In retrospect, the significance of the B♭-minor triad that hinted at a shift to an LPR loop about B♭, more likely lay in its inclusion of F, part of the anticipated G7.)

As the foregoing examples suggest, a neo-Riemannian perspective can offer insight into the organization of some triadic passages in neo-tonal works, and specifically can help to illuminate the relationship between foreground harmonic successions and middleground structures—whether these middleground structures project motivic content or anticipate functionally defined harmonic goals. Here we might add a caveat, however: the lack of emphasis on voice-leading parsimony in these examples does perhaps raise questions about the LPR model as a source of coherence—questions not raised as directly in tonal contexts, in which smooth voice leading remains normative. As in tonal contexts, we therefore encourage our students to consider passages such as these from various analytical perspectives. Salzer's extensions of Schenker's ideas (see footnote 13 above), for example, might equally well represent the structures identified here. If so, we would encourage our students to consider what one might convey by choosing one approach over the other. As such, and even though this kind of organization is by no means characteristic of all triadic passages in the neo-tonal repertoire, we find discussion of this approach, in conjunction with carefully chosen examples, useful in getting students to consider alternative approaches to neo-tonal organization.

In the context of a twentieth-century survey course emphasizing model composition, neo-Riemannian theory can also open doors to compositional work in neo-tonal styles. In addition to taking the above analytical examples as models—asking students to compose, for example, a non-functional triadic passage that projects a C-major triad in a way similar to that seen in the Salmenhaara passage—standard neo-Riemannian cycles can be treated as a compositional resource and manipulated to produce new successions. For example, we might take the RL cycle and reverse the mode of each triad, then instead of C-major—A-minor—F-major—we would get C-minor—A-major—F-minor and so on. We have taken the liberty

of recomposing the beginning of the Reger piece along these lines, as shown in Example 15 (note that the first chord is unaltered). The progression sounds somewhat strange, but not altogether foreign to Reger's style.



Example 15

Final Thoughts

The inclusion of neo-Riemannian theory in the undergraduate theory core brings to the fore important questions about the bases of tonal (and post-tonal and neo-tonal) coherence, but, as noted above, the approach's greatest pedagogical advantage lies in its capacity to support bridge building. From a purely pragmatic perspective, introduction of the neo-Riemannian approach not only reinforces and integrates what otherwise might seem to be disparate topics in chromatic harmony, but it also permits students to first encounter a transformational perspective on musical structure in a familiar setting, before confronting notions of pitch-class transformations and equivalencies. Somewhat more ideally, perhaps, we believe that this theoretical bridging encourages students to see continuities not only between tonal and post-tonal music theories, but also between nineteenth- and twentieth-century music.

A final and far from inconsequential benefit accrued from teaching the neo-Riemannian approach relates again to its bridging potential, in this case its potential to bridge the divide between contemporary research in music theory and the version of music theory that we typically teach to our undergraduate students. The frequent disconnect between what professional music theorists do and what we teach is widely recognized, of course, and neo-Riemannian theory's potential as a remedy has been noted elsewhere.[24] We believe that this opportunity to invite our students to enter into and to begin to explore our scholarly world, coupled with the other benefits outlined above, amply repays the investment of a few class periods in the study of neo-Riemannian theory.

[24] Cohn's (1998b) imagined theory curriculum, designed to address this concern, includes a unit devoted to voice leading between triads, taught in reference to the parsimonious LPR transformations and their *Tonnetz* representations. While Cohn's curriculum as a whole would be difficult to implement at most institutions, some of its components, including the neo-Riemannian approach, can be adopted into existing programs with relative ease.

Without specifically pointing to the inclusion of neo-Riemannian theory in this regard, Stephenson (2001) cites Roig-Francolí's efforts to incorporate professional methodologies as a strength of this text (see especially pp. 103 and 111).

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