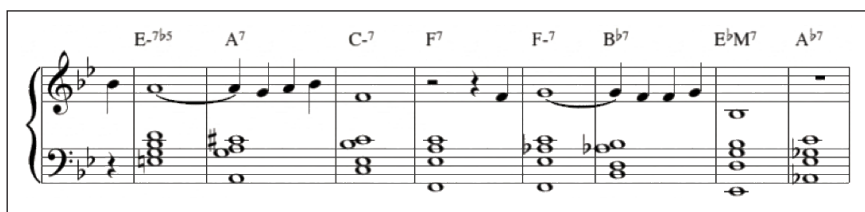


A Model of Common-Tone Connections Among Jazz Scales

STEFAN LOVE

One of the more difficult tasks confronting the jazz musician is the necessity of improvising over chord progressions that often imply many keys in quick succession. Even standards of the repertoire rarely employ chords from a single key only. The opening measures of “Stella By Starlight” are a familiar example:



Example 1: *Stella By Starlight*

The first two chords imply D minor, the next two B-flat major, and the last four present a ii-V-I cadence in the (non-tonic) key of E flat. To tackle this challenge, jazz instructors often teach their students to associate each chord type with several possible scales. (I use the term “scale” to refer to a pitch-class collection from which the student derives improvised material.) Before improvising, a beginning student labels each chord in a lead sheet with a particular scale, and then plays only notes from that scale at that point in the improvisation. Eventually, this process no longer requires advance planning: the student understands intuitively which scales are possible over a given chord and can choose among them without forethought. This association of chords with scales is often termed “chord-scale theory.”¹

Although students practice each scale type carefully to learn its shape and sonic effect, instruction on scalar connection is rare; in other words, a student with deep understanding of chord-scale theory might not consider how a scale relates to the scale that

¹ This concept underlies much jazz pedagogy. An early source is George Russell’s *The Lydian Chromatic Concept of Tonal Organization for Improvisation* (New York: Concept Publishing, 1959).

follows. Yet scalar connection profoundly affects the sound of an improvised solo. Independent of chord-scale theory, students often learn the importance of using common tones to link chords together. But seldom is this technique explicitly tied to the theory of scalar connection. In fact, the two are intimately related. In this paper I present a model of scalar connection applicable to jazz pedagogy and analysis.

A jazz handbook might present the Dorian scale beginning on D as a distinct scale, though it is identical in pc content with its “parent” scale, the major scale on C. (I use the term “parent scale” to refer to an *unordered* pc set.) Many other jazz scales are similarly redundant, and reducible to a smaller group of parents. Example 2 shows four common parent scale-types, transposed to begin on C: Diatonic, Melodic minor (degrees six and seven always raised), Harmonic minor, and Octatonic (half-whole).²



Example 2: Some parent scales

From left to right: Diatonic (D), Melodic minor (M), Harmonic minor (H), Octatonic (O)

The letters D, M, H, and O refer to these types, with a subscript indicating the *reference* pc (casually, “scale degree one”). For example, M3 designates the pcs of the melodic minor scale begun on E flat, *irrespective of order or rotation*. In total, there are 39 transpositions of parent scale-types considered here (12 Diatonic, 12 Melodic, 12 Harmonic, and 3 Octatonic).

² A fascinating discussion of the origins of these parent scales appears in Dmitri Tymoczko’s “The Consecutive-Semitone Constraint on Scalar Structure: A Link Between Impressionism and Jazz” (*Integral* 11 (1997): 135-179).

Maj7 _r	Dom7 _r	min7 _r	min7♭5 _r	minM7 _r	dim7 _r
D _r (Ionian)	D _{r+5} (Mixolydian)	D _{r+10} (Dorian)	M _{r+3} (Loc.nat.9)	M _r (Melodic)	O _{r+2} (W-H)
D _{r+7} (Lydian)	H _{r+5} (Mixolydian ♭13)	D _{r+3} (Aeolian)	D _{r+1} (Locrian)	H _r (Harmonic)	D _{r+8} (Mixolydian)
M _{r+9} (Lydian #5)	M _{r+5} (Mixolydian ♭13)	D _{r+8} (Phrygian)		D _{r+10} (Dorian)	D _{r+2} (Mixolydian)
	M _{r+7} (Lydian ♭7)	M _r (Melodic)		D _{r+3} (Aeolian)	D _{r+5} (Mixolydian)
	M _{r+1} (Altered)			D _{r+8} (Phrygian)	D _{r+11} (Mixolydian)
	O _r (Half-Whole)				

Table 1: Common jazz chord-types and associated scales, with root r (pc)

Table 1 converts chord symbols to parent scales. In effect, this is a distillation of the chord-scale theory described above. The heading of each column in the table gives a chord type, with the subscript “ r ” referring to the pc of the chordal root. Each entry in a column gives a possible scale associated with that chord, along with its conventional name (as it might appear in a chord-scale handbook) in parentheses. Consider E7, a dominant-seventh chord with root E. Refer to the “Dom7_r” column and let $r = 4$. The first possible parent scale is D₉, the Diatonic scale with A ($r + 5 = 9$) as reference pitch (i.e., “E Mixolydian”). Over E7, one could also use the notes of a harmonic minor scale with reference tone A ($r + 5 = 9$) (“Mixolydian ♭9, ♭13”), a melodic minor scale with reference tone B ($r + 7 = 11$) (“Mixolydian ♭13”), and so forth. I must reiterate that parent scales are best understood as unordered collections whose pitch classes are neutral with respect to function: the pitches of the parent scale D9 are treated differently when played over an E7 chord compared to when played over an Amaj7 chord.

Lead sheets often include extensions or alterations in their symbols: for example, a chord might be labeled “G7♭9#11.” This additional information would seem to imply certain scales and preclude the use of others. However, in practice, experienced players tend to treat these as suggestions only, and use whatever extensions or alterations they wish over a given chord. Therefore, I assume that one is reading a lead sheet for chord root and quality only, not extensions or alterations. My inclusion of the Dorian, Aeolian, and Phrygian scales over the minor-major seventh chord is similarly pragmatic: though the scale would seem directly to contradict the

chordal seventh, in certain styles, it is acceptable to preserve only the major or minor quality of the triad.

THE MODEL

Chord-scale theory becomes much more powerful when combined with consideration of common tones. In a solo built from scales sharing many common tones, the melody seems to glide over the shifting harmonies. On the other hand, a soloist could employ successive scales with *few* common tones for another effect, using the common tones as pivots. This technique can reinvigorate banal progressions: consider the incredible variety of scales often used over the ii-V-I cadence, the most clichéd progression of all. In the following section, I formalize the common-tone connections between the parent scales named above.

I define the relation between two scales as parsimonious if the scales share six common tones.³ In parsimonious relations between seven-note scales, the seventh pc is replaced by a pc a semitone away.⁴ Due to the differing cardinalities, relations between an octatonic scale and a seven-note scale are less elegant, though I retain the six-common-tone definition of parsimony. Table 2 shows all parsimonious scalar relations among the four types, and then generalizes them with respect to a root *r*.

³ For an analogous discussion of parsimony between triads, see Richard Cohn's "Neo-Riemannian Operations, Parsimonious Trichords, and Their 'Tonnetz' Representations" (*Journal of Music Theory* 41/1 (Spring 1997):1-66).

⁴ I borrow this careful phrasing ("replaced") from David Lewin's "Cohn Functions" (*Journal of Music Theory* 40/2 (Autumn 1996): 181-216). The obvious alternative formulation, in which a pc "moves" by semitone along the chromatic gamut, carries too many voice-leading connotations, which are unnecessary to my argument. The parsimonious relation between seven-note scales mirrors the single semitone-displacement of Cohn's "maximal smoothness" (Richard Cohn, "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," *Music Analysis* 15/1 (Mar., 1996): 9-40).

2.1: From Diatonic

	0	1	2	3	4	5	6	7	8	9	10	11			0	1	2	3	4	5	6	7	8	9	10	11
D ₀	C		C		C	C		C		C		X		D ₀	C		C		C	X		C		C		C
D ₅	C		C		C	C		C		C	X			D ₇	C		C		C		X	C		C		C

	0	1	2	3	4	5	6	7	8	9	10	11			0	1	2	3	4	5	6	7	8	9	10	11
D ₀	C		C		X	C		C		C				D ₀	X		C		C	C		C		C		C
M ₀	C		C	X		C		C		C		C		M ₂		X	C		C	C		C		C		C

	0	1	2	3	4	5	6	7	8	9	10	11
D ₀	C		C		C	C		X		C		C
H ₉	C		C		C	C			X	C		C

Generally: $D_r \rightarrow D_{r+5}, D_{r+7}, M_r, M_{r+2}, H_{r+9}$

Table 2: Parsimonious scalar connections
C = common pc, X = discarded/replaced pc

2.2: From Melodic

	0	1	2	3	4	5	6	7	8	9	10	11
M ₀	C		C	X		C		C		C		X
D ₀	C		C		X	C		C		C	X	

	0	1	2	3	4	5	6	7	8	9	10	11
M ₀	C		C	C		C		C		X		C
H ₀	C		C	C		C		C	X			C

Generally: $M_r \rightarrow D_r, D_{r+10}, H_r, O_{r+2}$

Table 2: Parsimonious scalar connections (continued)
C = common pc, X = discarded/replaced pc

2.3: From Harmonic													
	0	1	2	3	4	5	6	7	8	9	10	11	
H ₀	C	C	C	C		C		C	C			X	H ₀
D ₃	C		C	C		C		C	C			X	M ₆

	0	1	2	3	4	5	6	7	8	9	10	11	
H ₀	C		C	C		C		X	C			C	
O ₂	C		C	C		C	X		C	X		C	

Generally: $H_r \rightarrow D_{r+3}, M_{r3}, O_{r+2}$

2.4: From Octatonic													
	0	1	2	3	4	5	6	7	8	9	10	11	
O₀	C	C		C	C		C	X		X	C		O₀
M₁	C	C		C	C		C		X		C		H₁

Generally: $O_r \rightarrow M_{r+1}, M_{r+4}, M_{r+7}, M_{r+10}, H_{r+1}, H_{r+4}, H_{r+7}, H_{r+10}$

Table 2: Parsimonious scalar connections (continued)
C = common pc, X = discarded/replaced pc

The table compares pairs of parsimoniously related scales. Within a row, Cs and Xs designate pcs held in common or replaced between the two scales. Two examples should clarify the table: D_0 ("C major") connects to D_5 ("F major") by removing pc 11 and replacing it with 10. This generalizes to $D_r \rightarrow D_{r+5}$. Similarly, D_0 connects to M_2 ("D melodic minor") by discarding pc 0 and replacing it with 1. This generalizes to $D_r \rightarrow M_{r+2}$.

From the generalized parsimonious connections I derive my scale wreath, Figure 1.⁵ This Figure illustrates the connections between the 12 transpositions of the diatonic, melodic, and harmonic scales. For simplicity, I exclude the octatonic scale for now. The specific placement of the various scales is arbitrary; Table 2 dictates only the network of connections. Each node represents a particular transposition of one of the four parent scales; each line indicates a parsimonious connection (the different types of line are for visual clarity only). I refer to motion between connected nodes as a "step."

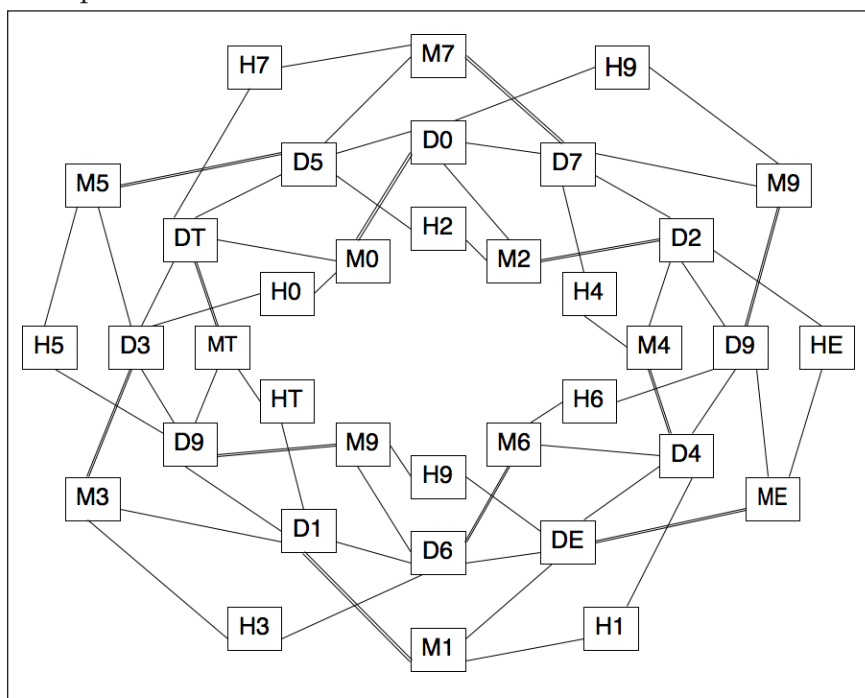


Figure 1: Scale wreath

⁵ Though his wreaths function differently, I adapted the idea of a wreath from Robert Morris, "Voice-Leading Spaces," *Music Theory Spectrum* 20/2 (Autumn 1998): 175-208.

The wreath consists of three concentric rings. The middle ring is the familiar circle of fifth-related diatonic scales. The outer and inner rings contain alternating harmonic and melodic scales; the outer ring's scales have odd-numbered roots, the inner, even. Though a single step always implies the loss of a single common tone, this property is inconsistent when making multiple steps.⁶ For example, scales four steps apart typically share three common tones, but sometimes share four. These exceptional cases manifest the effect shown in Table 3:

	0	1	2	3	4	5	6	7	8	9	10	11
M_0	X		C	C		C		X		X		C
D_{10}	~		~	~		~		~		~	~	
D_3	~		~	~		~		~	~		~	
M_3	~		~	~		~	~		~		~	
H_3			C	C		C	X		X		X	C!

Table 3: An exceptional relation

Common-tone replacement in one possible path from M_0 to H_3

A series of steps on the wreath can result in a common tone being lost and then regained, respelled enharmonically. In Table 3, the B-natural of C melodic minor (M_0) is replaced with C flat in E-flat harmonic minor (H_3), resulting in an unexpected common tone. Despite the respelling, the practical emphasis of my theory requires that I consider it a common tone: it would sound as such, on a keyboard at least, and it could function as a pivot between these scales. This exception prevents us from using the wreath as a *precise* guide to the common-tone relation between distant scales. However, the wreath remains accurate for all scales two steps apart, and most scales three and four steps apart. Relations between different scale types five or six steps distant *always* feature Table 3's exception. But scales far apart on the wreath, even if they have more common tones than they "should," will still sound distantly related. Therefore, the wreath presents a useful approximation of the closeness of scalar connection.

⁶ Consider motion from a scale all the way around the wreath back to itself: somewhere along the line, the common-tone mechanism must "flip."

The octatonic scale complicates matters somewhat. Table 4 presents the octatonic scale’s relations to scales with reference pcs 0, 1, and 2, with the others easily derived.

Relation	O/D0	O/D1	O/D2	O/M0	O/M1	O/M2	O/H0	O/H1	O/H2	O/O1	O/O2
CTS	4	5	5	4	6	4	3	6	5	4	4
Steps btw.	3	2	2	4	1	4	4	1	4	4	4

Table 4: Complete Octatonic relations

The octatonic scale’s only parsimonious relations are with the melodic and harmonic minor scales. Figure 2 shows the wreath with the addition of the three octatonic scales (see Table 2.4 for their parsimonious relations). The scale’s limited transpositions make this wreath less useful. For visual clarity, I include multiple nodes for each octatonic scale. It is tempting to view the multiple nodes as “wormholes” that allow one to move quickly from one end of the wreath to the other, with far fewer steps than before. For example, one might think that M_9 , in the upper right corner, is now only two steps from H_3 , in the opposite corner, via O_2 . But this reifies the wreath inappropriately, and leads away from the initial emphasis on common tones. When octatonic nodes are used as intermediate steps on a journey between other scale types, step relations on the wreath no longer correlate with common tones. Thus, when calculating scalar connection, the octatonic nodes should be considered only when they are endpoints of the journey.

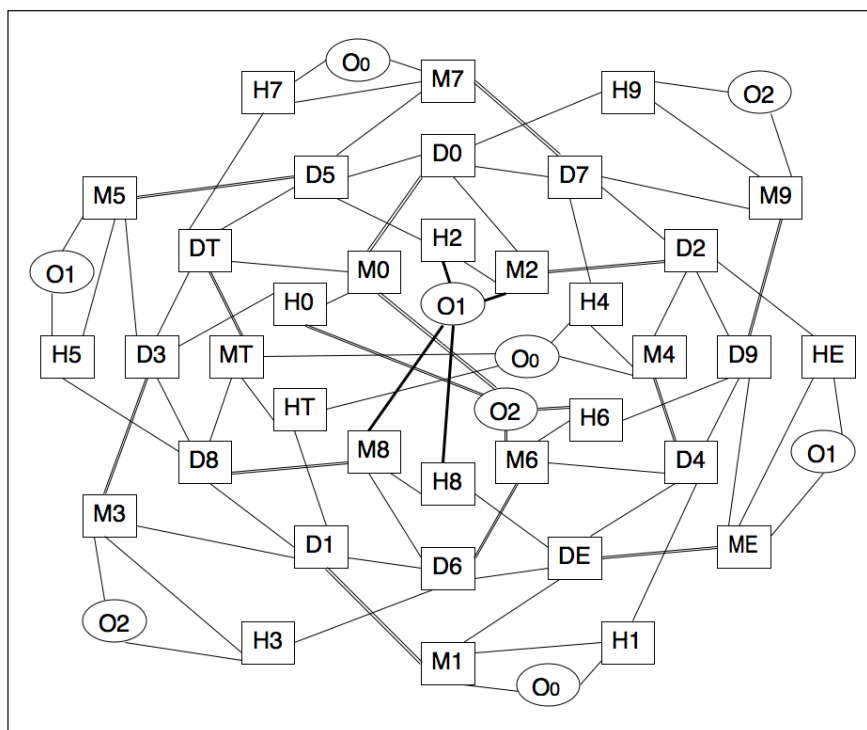


Figure 2: Scale wreath with octatonic included

APPLICATIONS

Used in conjunction with Table 1, the wreath in Figure 2 could be of great use to jazz students. It provides an excellent starting point from which to tackle a new tune. In this situation, jazz students are often trained to pick a scale for each chord before they begin playing, and to practice their improvisation using these scales; in the worst case, the students' scale choices are arbitrary, and the result sounds disjoint and "vertical." However, if they base their choices on the common-tone wreath, their chosen scales will reflect a conscious desire for close or distant connection. They will experiment with sounds based not on the superimposition of single scales over single chords, but rather on the connections between scales. The wreath illustrates that common-tone relations are not an isolated element of improvisation, but rather are closely tied to scale choice. Should a student experiment with particularly distant scales, the careful use of common tones becomes all the more important: the student must use the few common tones available if coherence is to be achieved.

To model this procedure, Table 5 presents possible scales for the opening eight bars of “Stella By Starlight” (Ex. 1), based on the chord-scale associations shown in Table 1.

<i>min7</i> _{b5} ₄	Dom7 ₉	<i>min7</i> ₀	Dom7 ₅	<i>min7</i> ₅	Dom7 ₁₀	Maj7 ₃	Dom7 ₈
M ₇	D ₂	<i>D</i> ₁₀	<i>D</i> ₁₀	<i>D</i> ₃	<i>D</i> ₃	<i>D</i> ₃	<i>D</i> ₁
<i>D</i> ₅	<i>H</i> ₂	D ₃	<i>H</i> ₁₀	<i>D</i> ₈	<i>H</i> ₃	D ₁₀	<i>H</i> ₁
	M ₂	<i>D</i> ₉	<i>M</i> ₁₀	<i>D</i> ₂	<i>M</i> ₃	<i>M</i> ₀	<i>M</i> ₁
	M ₄	<i>M</i> ₀	<i>M</i> ₀	M ₅	<i>M</i> ₅		<i>M</i> ₃
	M ₁₀		M ₆		M ₁₁		M ₉
	O ₀		O ₂		O ₁		O ₂

Table 5: Possible scales for “Stella”

Note: *italics* indicate scales from the “in” collection. **Bold** indicates the “out” collection (both begin with D₅).

The heading of each column lists the chord from the lead sheet (with subscript root), and the entries in each column present the possible scales, derived from Table 1. Using Figure 2’s wreath, I have chosen two sets of scales for this segment: one “in” set using closely related scales, and one “out” set using distantly related scales. Italicized scales represent the “in” collection, and bold scales are from the “out” collection.

Based on these collections, I composed two possible melodies over the opening chords of “Stella By Starlight” (Example 3). For the purposes of demonstration, I avoided non-scale tones and attempted to use many notes from each scale. Consequently, the passages sound a bit forced, but they should suggest the opposing effects of the two options. In identical fashion, a student could devise two complete sets of scales for a new tune and practice improvising with each set. Great benefits would be derived from alternating choruses of “in” and “out” improvisation.

3.1: "In"

3.2: "Out"

The image displays two musical examples, 3.1 and 3.2, each consisting of a piano (upper) and bass (lower) staff. Example 3.1, titled "In", shows a sequence of chords: E-7b5, A7, C-7, F7, F-7, Bb7, Eb, and Ab7. Example 3.2, titled "Out", shows the same sequence of chords. The notation includes various accidentals and note values, with the bass staff often featuring more complex rhythmic patterns and accidentals.

Example 3: Two renderings of "Stella"

Note: background scales are shown in lower staff

The procedure I describe above may seem laborious. Linking familiar scale-names (Dorian, Mixolydian, etc.) with the abstract parent scales of the wreath probably presents the largest intellectual hurdle. However, with experience, a student would begin to understand and manipulate the relations between scales with less forethought. Much jazz pedagogy is based on the premise that a student generalizes and unconsciously reapplies the lessons learned through laborious scale and chord analysis. The wreath and its associated procedures are a guide to the control of a particular improvisational variable – degree of common-tone scalar connection – of which the student can ultimately develop an internal, personal understanding. I believe that with practice, a student would come to understand and manipulate common-tone connections between scales intuitively.

The wreath is especially useful to determine smooth scalar connections across the unusual progressions typical of jazz from the '60s onwards (think of John Coltrane, Wayne Shorter, etc.). As an

example, consider the first eight bars of the bridge of “Very Early,” shown in Example 4.



Example 4: “Very Early” (bridge)

The unusual root motion of this chord progression requires extra attention to scalar connection to avoid calling attention to harmony at the expense of melody. Such a progression causes consternation to inexperienced players, who may be able to assign logical scales to each chord in isolation without knowing how to connect them. Furthermore, the scales that create smooth connections in this situation are far from obvious. The student’s goal is to find *some* set of scales that connect smoothly and facilitate melodic coherence.

Table 6 shows the full range of possible scales for this passage, derived from Table 1.

Bmaj7	Ab7	Dbmaj7	Bb7	Bmaj7	G7	Cmaj7	Ab7
D ₁₁	D ₁	D ₁	D ₃	D ₁₁	D ₀	D ₀	D ₁
D ₆	H ₁	D ₈	H ₃	D ₆	H ₀	D ₇	H ₁
M ₈	M ₁	M ₁₀	M ₃	M ₈	M ₀	M ₉	M ₁
	M ₃		M ₅		M ₂		M ₃
	M ₉		M ₁₁		M ₈		M ₉
	O ₂		O ₁		O ₁		O ₂

Table 6: Smooth scalar relations in “Very Early”

Note: solid lines between scales indicate parsimonious relation or identity; dotted indicate two-step relation.

Based on the Figure 2 wreath, solid lines between scales indicate a parsimonious relation (or identity), and dotted lines indicate a two-step relation. There are few smooth scalar paths leading through the entire progression, that is, paths using only parsimonious or two-step connections. Furthermore, the octatonic scale is the only scale over G7 that can connect Bmaj7 to Cmaj7. This is a perfect example of an

unintuitive scale choice. It is hard to imagine that a student assigning scales to this passage would stumble upon these paths. The difficulty of devising a convincing melody across these disjunct chords might frustrate the student a great deal. A better understanding of scalar connection would no doubt make easier the transition from “standards” to the more unusual progressions of later jazz.

Table 6 also reveals some of the patterns that the student would come to understand intuitively. Notice the isomorphic network of connections between BM7-G7 and CM7-A $\flat$ 7. Naturally, all progressions from a major-seventh chord to the dominant-seventh chord a major third below will feature analogous connections. Even these unusual chord connections would come to be understood through practice. A dedicated teacher could devise pages of exercises that included examples of every possible chord connection. The student who assigned scales to these progressions and practiced them diligently would be equipped to play over any modern jazz tune.

EXTENSIONS

The most natural extension to the theory is the inclusion of other parent scales, such as the harmonic major, whole-tone, and hexatonic. These latter two would share the octatonic’s awkwardness, but connect to many other scales. Additional scales could enrich the theory, though they would make the wreath more difficult to render and interpret. The theory could also be used to suggest parsimonious scalar shifts *within* a given harmony, as commonly occur in modal jazz.

My discussion of applications is focused on pedagogy rather than analysis, but the theory could serve well in an analytical role. Analysis might reveal that particular players tend to use certain scalar relations consistently. For example, over a ii-V-I progression, a player might favor close connection between ii and V, but distant connection between V and I – or vice versa. Along similar lines, the theory could reveal hidden consistencies in the chord progressions used by a particular composer. For example, a certain composer may favor adjacent chords that allow very few close scalar connections. More broadly, certain degrees of relation may be typical of certain style-periods, in both compositions and improvisations. Any jazz student or analyst will benefit from careful attention to the scalar relations modeled above.