Upon Further Reflection: Teaching Inversion through Jean Papineau-Couture's *Nuit*

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I. INITIAL CONSIDERATIONS

Pitch and pitch-class inversion is a core concept in the pedagogy of post-tonal music. There have been many theoretical and analytical studies on this topic, beginning half a century ago with the writings of Milton Babbitt, George Perle, and David Lewin.[1] These publications inspired many subsequent articles and model analyses of octatonic, atonal, and twelve-tone compositions.[2] This essay

[1] Babbitt outlines the inversional properties of symmetrically related twelve-tone rows in *The Musical Quarterly* 46/2 (1960), 245–59; "Twelve-Tone Rhythmic Structure and the Electronic Medium," *Perspectives of New Music* 1/1 (1962), 49–79; and "The Structure and Function of Music Theory," *College Music Symposium* 5 (1965), 49–60. Lewin describes a contextually-defined approach to inversion in "Inversional Balance as an Organizing Force in Schoenberg's Music and Thought," *Perspectives of New Music* 6 (1968), 1–21; "A Label-Free Development for 12-Pitch-Class Systems," *Journal of Music Theory* 21/1 (1977), 29–48; and *Musical Form and Transformations: 4 Analytic Essays* (New Haven: Yale University Press, 1993). Perle explores symmetrical properties in his own "twelve-tone tonal" works and the works of other composers, notably Berg, in "Symmetrical Formations in the String Quartets of Béla Bartók," *Music Review* 16 (1955), 300–12; *Serial Composition and Atonality* (Berkeley: University of California Press, 1962); and *Twelve-Tone Tonality* (Berkeley: University of California Press, 1962).

[2] Textbooks include Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); Stefan Kostka, *Materials and Techniques of Twentieth-Century Music*, 2nd ed. (Englewood Cliffs, NJ: Prentice Hall, 1998); Joel Lester, *Analytic Approaches to Twentieth-Century Music* (New York: W. W. Norton & Company, 1989); Robert Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987); John Rahn, *Basic Atonal Theory* (New York: Schirmer Books, 1980); Miguel Roig-Francolí, *Understanding Post-Tonal Music* (New York: McGraw-Hill Higher Education, 2008); Joseph N. Straus, *Introduction to Post-Tonal Theory*, 3rd ed. (New Jersey: Pearson Education Inc., 2005); and J. Kent Williams, *Theories and Analysis of Twentieth-Century Music* (Orlando, FL: Harcourt Brace & Company,

outlines a self-contained module on axial symmetry that is intended to culminate with an analysis of *Nuit*, a solo piano composition by Jean Papineau-Couture.[3] The module, which could be part of second-year undergraduate course, is designed to teach students to hear, recognize, and model the properties of symmetrical designs.[4] Several factors make *Nuit* a strong teaching piece, among them intriguing timbres, with striking and strumming on the strings[5],

1997). Analyses include Brian Alegant, "When Even Becomes Odd: A Partitional Approach to Inversion," *Journal of Music Theory* 43/2 (1999), 193–230; Jonathan Bernard, "Space and Symmetry in Bartók," *Journal of Music Theory* 30/2 (1986), 185–201; Michael Cherlin, "Dramaturgy and Mirror Imagery in Schönberg's *Moses und Aron*: Two Paradigmatic Interval Palindromes," *Perspectives of New Music* 29/2 (1991), 50–71; Lora L. Gingerich, "Explaining Inversion in Twentieth-Century Theory," *Journal of Music Theory Pedagogy* 3/2 (1989), 233–42; Philip Lambert, "On Contextual Transformations," *Perspectives of New Music* 38/1 (2000), 45–76; Stephen Peles, "'Ist Alles Eins': Schoenberg and Symmetry," *Music Theory Spectrum* 26/1 (2004), 57–86; Larry J. Solomon, "New Symmetric Transformations," *Perspectives of New Music* 11/2 (1973), 257–64; and Philip Stoecker, "Studies in Post-Tonal Symmetry: A Transformational Approach" (PhD diss., City University of New York, 2003).

[3] Papineau-Couture (1916–2000) was a Canadian composer who spent the majority of his life in Montreal.

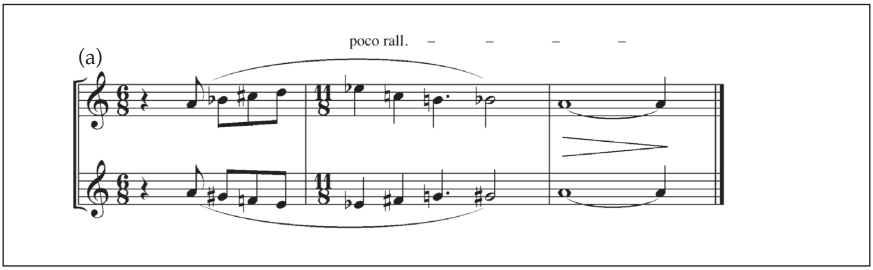
[4] Our assumptions are that the undergraduate course would meet three times a week for an hour while a graduate analysis course would meet once a week for two hours and twenty minutes. Ideally, the undergraduate course would be joined with an aural skills class devoted to post-tonal music whereas the graduate course might incorporate ear training in the manner of Michael Friedmann's *Ear Training for Twentieth-Century Music* (New Haven: Yale University Press, 1990). Additionally, we have chosen not to include biographical or historical background on the composers we discuss for two reasons: we are concerned primarily with the theoretical/analytical angle of inversion, and this information is readily obtainable through on-line databases (and Wikipedia of course). We should also point out that symmetrical designs are not a 20th-century invention; we can find palindromes and inversional designs in Renaissance and Classical works. See Robert P. Morgan, "Symmetrical Form and Common-Practice Tonality," *Music Theory Spectrum* 20/1 (1998), 1–47, for a treatment of symmetrical structures and palindromes in common-practice music.

[5] According to Papineau-Couture, "[i]n it I have made use of various techniques of vertical symmetry but the structure is extremely free. My challenge was to integrate with the normal playing on the keyboard the many diverse timbres of direct striking on the strings, this in a

and an inventive and sophisticated set of inversional techniques. In addition, it is an extended work that lasts approximately 16 minutes, and thus rewards a variety of approaches. Below, we review four basic types of pitch and pitch-class symmetries, and briefly define even and odd index numbers. We then illustrate the common axial techniques from the literature, and show how these techniques inform various organizational levels of the work.

II. Four types of inversional designs

Example 1(a) shows the conclusion of the first movement of Bartók's *Music for Strings, Percussion, and Celeste*.[6] This brief codetta illustrates what is perhaps the clearest inversional design for students to recognize aurally and visually: *vertical symmetry*. The two melodic lines begin on a unison A4, wedge outward by a tritone, and return to A4, which functions as a *pitch axis*. Each pitch above A4 occurs with its inversional partner in a one-to-one correspondence. Here, all twelve pitch-classes are accounted for. Using a system where pitch-class C = 0, the pitch classes of the upper and lower lines sum to index number 6.[7]



Example 1(a). Conclusion of the first movement of Bartók's *Music for Strings, Percussion, and Celeste.*

homogenous and non-stop discourse. The form was spontaneously born from this marriage of colours." Véronique Robert, *Jean Papineau-Couture*, Centredisques CMCCD 6499.

[6] Bernard, "Space and Symmetry in Bartók," 186–88, discusses this passage and the large-scale symmetrical design of the pitch and pitch-class axes in this piece.

[7] The term "index" number first appears in Babbitt's "Twelve-Tone Rhythmic Structure": " … the sums of the pitch numbers of each pair are equal. This unique (for each equivalence class) number is called the 'index' of the equivalence class." (57)

As is well known, an even index number partitions the total chromatic, or *aggregate*, into five different pitch-class pairs—in this case, G♯/B♭, G/B, F♯/C, F/C♯, and E/D—and two pitch-class singletons that map *into* themselves (i.e., A/A and E♭/E♭). These axes are equal to half of the index number and half of the index number plus six. The singletons in any even index number are related by a tritone. Harmonically, even index numbers produce only even interval classes—0, 2, 4, or 6. If the index number is odd, the axes occur between two pairs of notes. For example, consider the index number 9. 9 divided by two is 4.5; thus the axes lie between 4 and 5, and t and e. Example 1(b) shows a realization of vertical symmetry with an odd index number, 11; the excerpt is the conclusion of Schoenberg's Piano Concerto, op. 42. Odd index numbers generate six different pitch-class pairs and no singletons, which is to say that no pitch class is able to map into itself. Harmonically, odd index numbers generate only odd interval classes—1, 3, or 5.[8]

(b)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P3: < 3, t, 2, 5, 4, 0,  6, 8, 1, 9, e, 7 >
I8: < 8, 1, 9, 6, 7, e,  5, 3, t, 2, 0, 4 >
Index number: 11, 11, 11, ...
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Example 1(b). Conclusion of Schoenberg's Piano Concerto, op. 42, iv.

[8] Babbitt, "Twelve-Tone Invariants," 254, discusses the intervallic types that result from either an odd or even index number. For a more recent discussion of odd and even textures (analytically and aurally) see Alegant, "When Even Becomes Odd."

We use the term *inversional canon* to describe a vertical design that is "asynchronous," which is to say that it is not presented in note-against-note fashion. With inversional canons, one melodic line leads while its reflection follows.[9] The voices can be close together (an eighth or sixteenth note apart) or quite remote (at a distance of five, ten, or more measures). Example 2 shows an inversional canon from Bartók's "Subject and Reflection." In mm. 47–50 the right and left hands mirror each other about an imaginary C♯5 axis and project index number 2. The left-hand is consistently an eighth

Overview of the even index numbers

Section:	I	II	III	IV	V	VI	VII
Index number:	8	10	4	6	0	2	8
Transpositional "move":	+2	+6	+2	+6	+2	+6 (which takes us back "home")	

Example 2. An inversional canon from Bartók's "Subject and Reflection."

[9] Cherlin, "Dramaturgy and Mirror Imagery," 52–53, uses the term "successive mirrors" to describe what we call inversional canons.

note behind the right-hand, realizing an inversional canon. In m. 63, another inversional canon occurs, but now the left hand is delayed two eighth notes, and the index number is 8.[10]

"Subject and Reflection" is an excellent stepping stone for *Nuit*. Bartók uses *every* even index number in the work. He does so by transposing the index numbers by <+2> and then <+6> until he reaches the original setting, which we might construe as "home." Two other structural features of "Subject and Reflection" are worth noting. First, the work occasionally deviates from strict inversional canons; for instance, in mm. 63–66, the left hand's E3 and A3 can be perceived as "dissonant" notes that disrupt or offset the mirror symmetry.[11] Second, Bartók periodically moves from strict one-to-one vertical correspondence to inversional canons, and back. Measures 23–25 and 40–42 illustrate two (of many) instances of changing alignment; such deviations are easily heard.

Palindromic symmetry is another important technique—especially in twelve-tone music. An inversional design with palindromic symmetry is a "temporal, horizontal" structure: it is organized around a midpoint on a horizontal plane, or time line, so that the first note is paired with the last note, the second note is paired with the penultimate note, and so on. Composers frequently employ two types of palindromes: pure retrogrades, which are R-symmetrical, and retrograde inversions, which are RI-symmetrical.[12] Example 3 provides an intriguing instance of horizontal symmetry in Luigi

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[10] Bernard, "Space and Symmetry," 188–91, addresses the large-scale symmetrical design of the pitch and pitch-class axes in this piece.

[11] The piece provides students with opportunities to hear "modulations" among and between inversional centers. Bartók uses different tempi to help listeners hear the progression of axes that unfold in the piece. In addition, the piece presents occasions to hear, model, and discuss disruptions of rhythmic canons and "pure" inversional designs. Such disruptions create an opportunity for students to notate the "correct" inversional partner for the right and left hands.

[12] Cherlin, "Dramaturgy and Mirror Imagery," 53, provides an extensive discussion of this mirror type. He calls this an interval palindrome, which "focuses upon the symmetrical display of ordered pitch intervals (or ordered pitch-class intervals, as the case might be) as they radiate out from some central axis." He then states that this mirror type "is normally the most difficult to perceive … ."

The two vocal presentations of "Acheronte" are palindromes with respect to their durations and their interval class profiles:

< E♭, D, G♯, B, A >	< F♯, E, G, D♭, C >
ics: 1, 6, 3, 2	2, 3, 6, 1

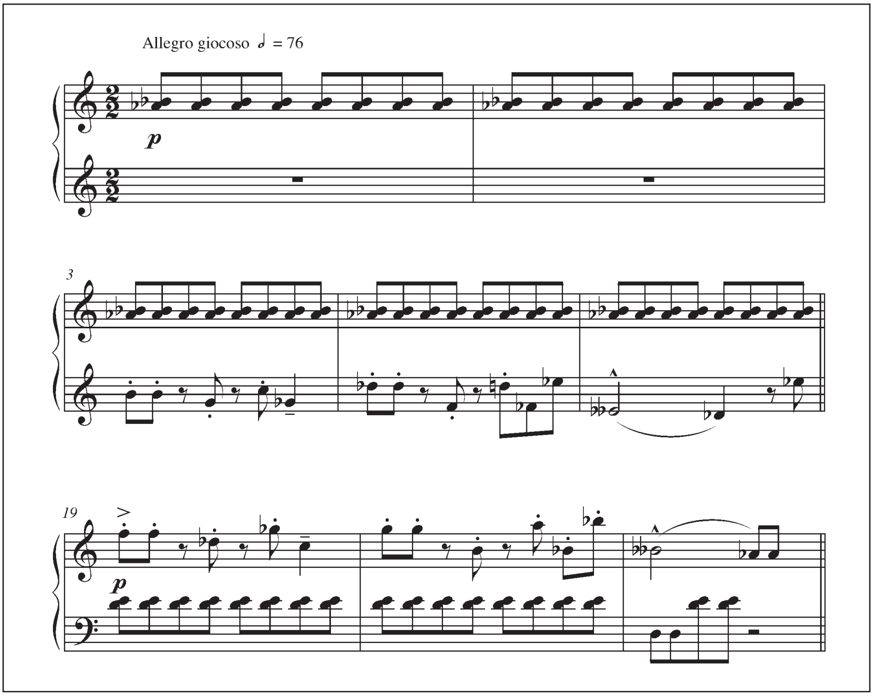
Example 3. Horizontal (R1) symmetry in Dallapiccola's, *Cinque canti*, iii (1955-56)

Dallapiccola's *Cinque canti*. This cruciform structure is based on a row that is RI-symmetrical, which is to say that the row can be mapped onto itself under retrograde and inversion (under an odd index number).[13] The back-to-back statements of "Acheronte" create a palindrome with respect to duration, meter, and melodic profile. This palindromic symmetry is reflected in the voice's pentachords, as the interval class succession <1, 6, 3, 2> is answered by <2, 3, 6, 1>.

In addition to vertical symmetry, inversional canon, and horizontal symmetry, students need to recognize a fourth type of symmetry: the *spiral*. A spiral is a configuration in which the inversional pairings between two lines alternate back-and-forth in register, creating a compound melodic line that incrementally wedges inward or outward. Example 4 highlights two inversional spirals from Bartók's Bagatelle, op. 6, no. 2. Beginning with an A♭4/B♭4 dyad, the music in mm. 1–5 wedges outward, alternating between pitches above and below the fixed dyad. In m. 3, B4, which lies a semitone above the A♭4/B♭4 dyad, is answered by its partner G4, which lies a semitone below. The music then leaps to C5, two semitones above the fixed dyad, which is then answered by G♭4, and so on. We would want students to note three other aspects of the passage. First, the A4 axis is conspicuously absent. This is an important observation, because an axis need not be present (or audible) in a symmetrical context. Second, the leap to the half note E𝄫4 in m. 5 disrupts the symmetry: the E♭5 at the end of m. 4 is not immediately followed by its inversional partner, E♭4; instead the spiral overshoots to E𝄫4, which receives the duration of a half note.[14] (We would argue that this substitution of E𝄫4 for E♭4 has far-

[13] Dallapiccola referred to this design as an "ideogram." He explained: "It is probably the case that I had chosen a series of this kind because of the need that I felt to draw the Cross on the score in musical notes, and at the same time to be able to show graphically the idea of the arms attached to the Cross by means of two other lines." Variations of this ideogram appear five times in the central song, which serves as the dramatic core of the cycle. It is also worth noting that Dallapiccola insisted that these ideograms be reproduced faithfully in the score. This has been noted by many scholars, including Ann Phillips Basart, "The Twelve-tone Compositions of Luigi Dallapiccola" (PhD diss., University of California, 1960); Roberto Zanetti, *La musica italiana nel novencento* (Busto Arsizio: Bramante, 1985), 1174, n. 44; and Rosemary Brown, "Continuity and Recurrence in the Creative Development of Luigi Dallapiccola" (PhD diss., University of Wales, 1977).

[14] It would be worthwhile to explore the role of the "missing" E♭4 here, discussing not only how Bartók saves this pitch for a later moment, but also addressing when it actually arrives and how it relates to the form of the piece. Another avenue of discussion would be to talk about



Example 4. Two inversional spirals from Bartók's Bagatelle, op. 6, no. 2.

reaching implications for the deep-level pitch organization, which offers many subtleties upon closer inspection.) Third, the pitches of the right hand in mm. 19–21 are now symmetrically organized about E♭5 while the left hand D4/E4 dyad is symmetrically spaced about E♭4. Though the pitch axis has shifted a tritone away—to both E♭5 and E♭4—the index number is still 6.

the rhythmic structure of the opening, or of more of the piece; for instance, one might imagine that m. 4 could be a basic idea repeated, leading to a sentence, but this realization is thwarted. A good model for a similar (analytical) application of pulse streams in a syncopated *Mikrokosmos* piece appears in John Roeder, "Rhythmic Process and Form in Bartók's 'Syncopation'," *College Music Symposium* 44 (2004), 43–57.

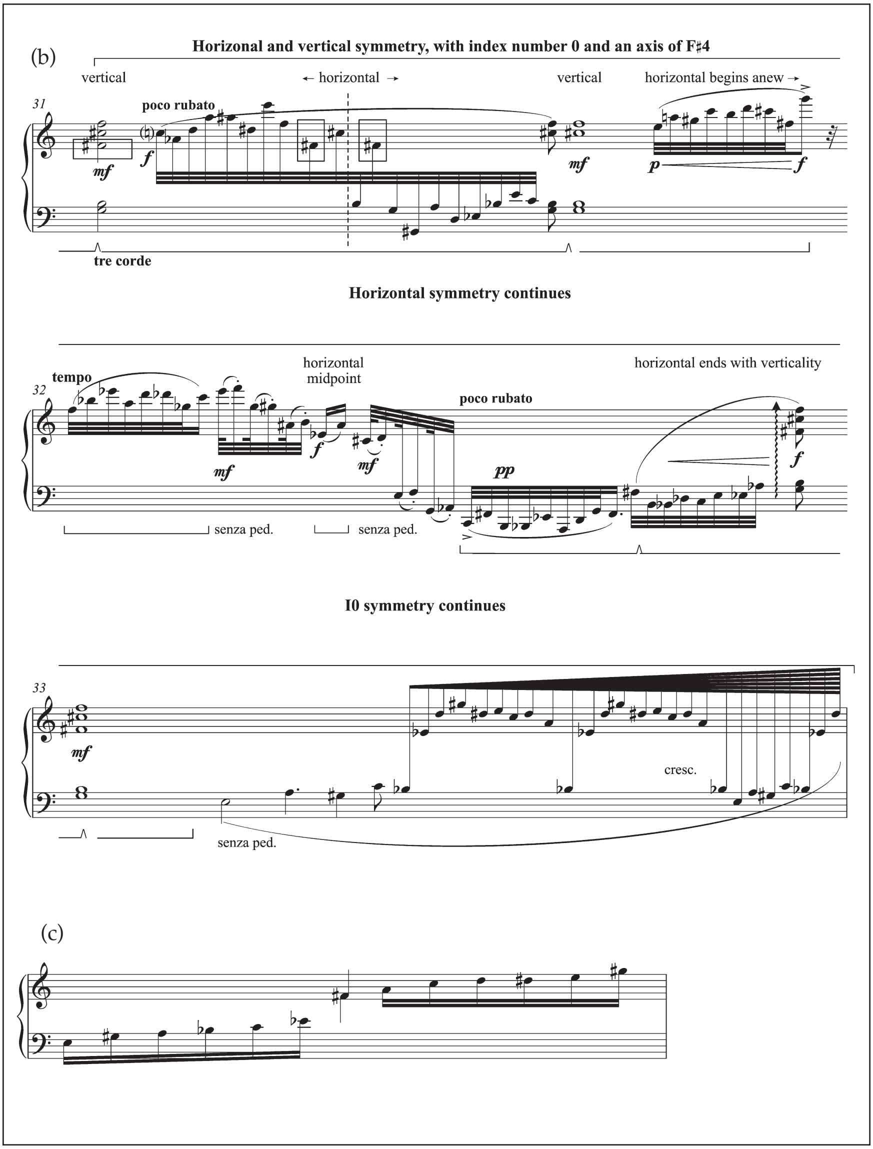
III. Some observations on *Nuit*

Once students have become familiar with the four basic types of symmetrical designs and have explored longer, more challenging works in order to gain practice hearing, identifying, and modeling inversional settings (see Section IV of this article), they are ready to explore *Nuit*'s abundance of palindromic, vertical, and spiral designs that are governed by even and odd index numbers. As a detailed analysis of this work would far exceed the scope of this essay, we will examine a few excerpts to suggest the variety of symmetrical designs and to detail the structural shifts in axes. Those looking to engage more closely with the axial landscape will find that entire passages are inverted around different axes, bringing to light multiple, simultaneous points of reference.

The excerpt in Example 5 is emblematic: it features all four types of symmetry—vertical (both one-to-one and canonic), palindromic, and spiral—and it realizes these symmetrical designs with both even and odd index numbers (specifically 10, 7, and 0). This passage includes distinctive transitions among inversional settings that shift axes. Indeed, Papineau-Couture employs many strategies for moving among different pitch axes. One strategy involves single notes or dyadic "pivots" that provide gateways into new inversional settings (just as a pivot chord in tonal music can function simultaneously in two keys.) For instance, m. 27 projects a spiral design that is governed by the index number 10; the pitch axis is F4. The pitch realization highlights the pitch-class invariance—retained pitches or pitch classes after transpositional or inversional operations—between the initial gestures of the right and left hands, which share a (012) trichord, {A♯, B, C}. The tempo of the spiral begins to slow upon the last iteration of A♯1; there follows an even whole-tone collection (the one that includes the even-numbered pitch classes: A♯1, C2, E2, F♯5, D3, G♯3) and a subset of the odd collection (the one that includes the odd-numbered pitch classes: C♯4, A4, G4, D♯4, F4). In m. 28, after the spiral finally attains the F4 axis, it is immediately followed not by D♯4 and G4, the expected partners under index number 10, but rather by D♯4 and E5. The tremolo, then, effectively modulates to index number 7. The shift from an even to an odd index number is striking, because the whole-tone sonorities in the inversional spiral are banished in favor of "crunchier" odd intervals comprising 13, 7, and 1 semitones (the midpoint A4/B♭4).

Example 5(a). *Nuit*, mm. 27–30

A similar modulation occurs in m. 29. The music at the beginning of m. 29 is governed by index number 7. After the second fermata, we have every reason to expect that the repeated C♯5 in the right hand will be followed by F♯4; instead it is answered by B3, which modulates to index number 0. From another perspective, F♯4, the



Examples 5(b). *Nuit*, mm. 31–33; and 5(c), *Nuit*. Pitch summary of the acceleration in m. 33, situated about the F♯4 axis.

lower note of the fermata, becomes the new axis, while C♯5, the higher note, initiates a new idea. (The change in tempo is also significant: specific tempos are matched to distinct index numbers and thus different inversional designs.)

Example 6. The culmination of *Nuit*.

The culmination of *Nuit,* shown in Example 6, weaves together a number of previous configurations. The pitch material in mm. 46–48 (reproduced on the bottom staff) and mm. 49–51 (on the top staff) is reflected about an F♯4 axis, thus enforcing a large-scale projection of index number 0. The passage features four instances of vertical symmetry governed by odd index numbers (7 and 5) and two instances of palindromic symmetry governed by index number 0. In addition, this passage contains brief sections of free, atonal music. For example, the transitional material between indexes 7 and 5 unfolds three trichords, each a member of set class (016). Analysis of these three collections reveals that the second (016) trichord is inversionally (not transpositionally) related to the first and third under T_2I and T_5I respectively. The strumming on the strings at the end of m. 49 (notated with squiggly lines) signals that the pitch material in mm. 46–48 will now be reflected about F♯4, beginning with index 7 in m. 49. (As an aside, strumming is routinely used in the work to "re-set" the pitch surface and erase or obscure pitch axes. This mode of articulation thus serves as a formal marker, or punctuation device.)

There is a great deal more to say about this intriguing and intricate composition, including the use of derived aggregates, (ordered) twelve-tone strings, and systematic relationships among timbres and rhythms. One might use *Nuit* as the focus of a midterm or final project. We might ask students to choose a brief passage from the piece and explore it in depth or write on the variety of inversional patterns; or even write a "model composition" that exploits the four types of symmetries. We might also ask students to create a *road map* of the work that would annotate what they see and hear in the work. The road map might be literal or abstract; it might focus on gestures, pitch and pitch classes, or axes; in short, a road map could take on a number of forms.

Example 7 offers such a road map of *Nuit*, one that highlights some of its salient characteristics. This reduction reveals associations among vertical, palindromic, and spiral configurations. It also provides an overview of the underlying pitch structure, which covers the entire range of the keyboard. On a first or second hearing the pitch structure and the proliferation of axial designs might seem bewildering and complex. However, repeated hearings (reinforced by a close reading of the score) reveal just a handful of symmetrical axes, chief among them repetitions about F4 (mm. 22–27 and 34–35) and F♯4 (mm. 29–34 and 46–50), and collisions of A4/B♭4 (mm. 2–9, 21, 28–29, 49, and 51). Such changes in the background axial organization signal important melodic, harmonic, and formal events. Notice, for instance, that the music at the very end of *Nuit*, mm. 50–51, flashes back to the first two measures (and the initial axis of symmetry, too). In a sense, the strumming at the end of m. 50 and the vertical symmetry in m. 51 create a structural frame: these two gestures appear at both the beginning and the end of the piece. This framing technique is a deep-level manifestation of horizontal symmetry.

A deeper analysis of *Nuit* will uncover many variants of the procedures just discussed, some obvious and others subtle. To explore further the counterpoint of inversional designs an instructor might adopt a "pair, share, and compare" strategy. First, divide the piece into discrete sections, parse the class into groups, and encourage each group to explore a section in as much detail as possible. Then re-convene the entire class, and ask one member from each group to join another group so that each person takes turns teaching his or her section to the rest of the students. A second class (or third) could then ask the original (or newly formed) groups to put the pieces together and find deeper connections among formal elements and, of course, inversional designs.

Example 7. A "road map" of *Nuit*.

IV. Writing/Analytical Exercises

Example 8 lists a handful of octatonic, atonal, and twelve-tone compositions that are well suited for a module on inversion. This list represents just a fraction of the symmetrical repertoire, of course. Some of the works exhibit transparent realizations of inversional symmetry; others are rather complex and could certainly be the focus of several class meetings. We endorse these pieces for several reasons: we have used them with a degree of success, scores and recordings are relatively easy to obtain, and we are confident that they will provide students with the skills to confront *Nuit* (or another composition worthy of a capstone project).

Vertical symmetry. Bartók's *Mikrokosmos* contains many instances of vertical symmetry in note-against-note and canonic formations. Suitable candidates include:

# 105 ("Game," mm. 10–18)	easy
# 109 ("Isle of Bali")	easy
# 142 ("From the Diary of a Fly")	medium
# 141 ("Subject and Reflection")	medium
# 144 ("Minor 2nds, Major 7ths")	medium
# 140 ("Free Variations")	harder

More extensive inversional designs can be found in Bartók's larger pieces, such as *Music for Strings, Percussion, and Celesta*, the Fourth and Fifth String Quartets, the scherzo movement from the *Suite*, op. 14, and the Sonata for Two Pianos and Percussion. "Song of the Harvest," for two violas, features a straightforward use of vertical symmetry with invertible counterpoint in an octatonic context. See also Webern's Canons, op. 16. Analyses of these pieces include (among others): Elliott Antokoletz, *The Music of Béla Bartók: A Study of Tonality and Progression in Twentieth-Century Music* (Berkeley: University of California Press, 1984), Antokoletz, *Twentieth-Century Music* (Englewood Cliffs, NJ: Prentice Hall, 1996); Jonathan Bernard, "Space and Symmetry in Bartók," *Journal of Music Theory* 30/2 (1986), 185–201; Richard Cohn, "Inversional Symmetry and Transpositional Combination in Bartók," *Music Theory Spectrum* 10 (1988), 19–42; Edward Gollin, "Some Unusual Transformations in Bartók's 'Minor 2nds, Major 7ths'," *Intégral* (1998), 25–51; Robert Katz, "Symmetrical Balancing of Local Axes of Symmetry in Bartók's Divided Arpeggios," *International Journal of Musicology* 2 (1993), 333–347; George Perle, "Symmetrical Formations in the String Quartets of Béla Bartók," *Music Review* 16 (1955), 300–312; and Leo Treitler, "Harmonic Procedures in the Fourth Quartet of Béla Bartók," *Journal of Music Theory* 3/2 (1959), 292–298.

Example 8. Examples of symmetry from the literature. *(continued next page)*

Inversional canons in twelve-tone settings:

Webern, Op. 27/ii	easy
Dallapiccola, *Quaderno musicale di Annalibera*, iii and v	easy
Carter, *Canon for 3*	medium
Webern, Op. 21, i and Op. 22, i	medium

Representative analyses include Brian Alegant, *The Twelve-Tone Music of Luigi Dallapiccola* (Rochester, NY: University of Rochester Press, 2010); Kathryn Bailey, "The Twelve-Note Music." *Canadian University Music Review* 9/1 (1988), 83–103, Bailey, *The Twelve-Note Music of Anton Webern: Old Forms in a New Language* (Cambridge: Cambridge University Press, 1991; Thomas DeLio, "Spatial Design in Elliott Carter's *Canon for 3*," *Indiana Theory Review* 4/1 (1980), 1–12; Andrew Mead, "Webern, Tradition, and 'Composing with Twelve Tones'," *Music Theory Spectrum* 15/2 (1993), 173–204; Catherine Nolan, "Structural Levels and Twelve-Tone Music: A Revisionist Analysis of the Second Movement of Webern's Piano Variations opus 27," *Journal of Music Theory* 39/1 (1995), 47–76; and Peter Westergaard, "Webern and 'Total Organization': An Analysis of the Second Movement of Piano Variations, Op. 27," *Perspectives of New Music* 1/2 (1963), 107–120.

Palindromic (retrograde and retrograde-inversional) symmetry is found in:

Webern, Op. 27, i	easy
Dallapiccola, *Goethe-Lieder*, ii	easy
Crumb, *Madrigals*, Book IV, i ("¿Por qué nací …")	easy
Dallapiccola, *Sex carmina alciae*, i, ii	easy
Webern, Op. 21, i (middle section), ii (all)	medium
Babbitt, *The Widow's Lament in Springtime* (vocal lines)	difficult

More extended examples of palindromes are found in Berg's *Lulu* (mm. 656–718, the heart of the "film" scene); *Der Wein* (mm. 112–172); and (though less strict) the *Lyric Suite*, iii, and Chamber Concerto, ii. Another extended instance of palindromes is found in Dallapiccola's *Commiato*, whose first and fifth and second and fourth movements are near exact palindromes with respect to pitch and rhythm. See Thomas DeLio, "Spatial Design in Elliott Carter's *Canon for 3*," *Indiana Theory Review* 4/1 (1980), 1–12; Catherine Nolan, "New Issues in the Analysis of Webern's 12-Tone Music," *Canadian University Music Review* 9/1 (1988), 83–103; George Perle, *The Operas of Alban Berg: II:* Lulu (Berkeley: University of California Press, 1985), Perle, *Style and Idea in the Lyric Suite*, 2nd ed. (Hillsdale, NY: Pendragon, 2001); and Mark Starr, "Webern's Palindrome," *Perspectives of New Music* 8/2 (1970), 172–42.

Example 8. Examples of symmetry from the literature. *(continued)*

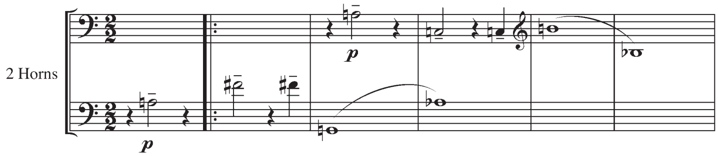
As a supplement to the pieces listed in Example 8, the brief exercises in Example 9(a) can help students explore various realizations of inversional symmetry. To begin, we might ask students to identify the inversional type (spiral, palindrome, etc.), label the index number, show the pitch-class mappings, and specify the pitch and pitch-class axes. We might then ask them to re-notate each canonic passage as a note-against-note design, and to realize the design on the keyboard or another instrument. They could then transform each design from an even context to an odd one, or vice versa, to hear (and feel) the latent harmonic differences of even and odd configurations. Then we might ask students to react to the surface: Is the axis present? How is it emphasized? Are there irregularities in the realization of the design? What do you *hear?* And finally, a bit of speculation: based upon your understanding of the opening, what might you expect to happen in the remainder of the movement?

Once students are comfortable analyzing inversional settings, they can begin to create their own inversional designs. Initially, we would give them fragments or melodies, such as those shown in Example 9(b), which they could manipulate under even and odd index numbers, in staggered imitation, in note-against-note formations and inversional canons.[15] Eventually they could create their own designs and model compositions from scratch, and have them perform their realizations on the piano or another instrument. These performances can be performed on pairs of like instrumentation (e.g., strings) and unlike instrumentation (e.g., oboe and cello). A performance with varied instrumentation would work best, for students can then hear the different timbres and effects. More ambitious students could fashion palindromic melodies and utilize different inversional contexts, rather than employing a single index number for the entire piece. Such activities would provide many opportunities for feedback and discussion.

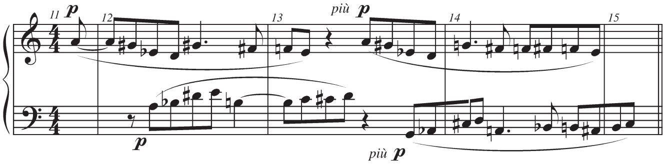
[15] Having students successfully create spiral designs from a given melody depends on the structure of the melody; thus this exercise is more abstract in design. For instance, we would ask students to create an inward or outward spiral based on an even or an odd index number. We would then instruct them to freely swap any notes with its neighbor. So an outward index-2 spiral could jump back and forth from <1 (axis) 2 0 3 e 4 t 5 9 6 8 7> to <1 (axis) 0 2 e 3 4 5 t 9 8 6 7>. Students could realize this design in pitch space, either "rhythmicized" or in even note values. They could then harmonize each note, or pairs of notes, of the scale with trichords or other collections.

(a) For the following passages, please provide the axis of symmetry (there may be more than one), the index number, and the inversional type (vertical, horizontal, spiral, or inversional canon). Also, write a brief response to each passage. You can focus on rhythm, texture, and the presence (or not) of the axis pitch.

Webern, *Symphonie*, op. 21, first movement, mm. 1–6



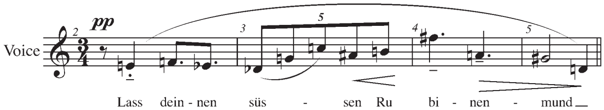
Bartók, "Chromatic Invention," Mikrokosmos, no. 91, mm. 11–15



Perle, *Modal Suite (mm. 5–6 and 11–12)*

(b) For the following melodies, create your own compositional designs, manipulating these tunes under even and odd index numbers, in staggered imitation (inversional canons), note-against-note formations, and spirals.

Dallapiccola, *Sex Carmina Alcaei*, mm. 2–5



Schoenberg, String Quartet no. 4, first movement, mm. 1–2

Examples 9(a) and 9(b). Exercises to reinforce inversional concepts.

We hope to have achieved two aims with this essay. The first is to give instructors a head start on devising a self-contained unit on inversion. These exercises will help students learn to successfully hear, identify, and write about the different types of inversional contexts that they are likely to encounter in the literature. By the time students create their own inversionally organized realizations, they will be able to analyze the varied inversional contexts in *Nuit*. Our second goal is to further the conversation on teaching inversion in post-tonal music and to suggest that *Nuit*, with its bewildering variety of symmetrical designs that interact on local and large-scale levels of organization, is indeed a composition worthy of study and reflection.

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