Three Exercises Involving Goldberg Canons

The following worksheets provide improvisation, singing, analytic, and compositional exercises involving canons from Bach's *Goldberg Variations* (or *Aria mit 30 Veränderungen*, BWV 988), published in 1741. They are designed as supplemental activities for second-year theory classes (which typically do not include units on counterpoint or canon), although they could also be used in counterpoint courses, where they might themselves be supplemented with further dictation, improvisation, partimento, and other structured composition exercises.[1] Currently, I use them in my third-semester theory class in an effort to expose students to (and stimulate interest in) the notion of strict counterpoint. They are part of an unofficial unit on my favorite canons.

The exercises below introduce students to some of the important aspects of canons that may have escaped their notice prior to this point in the curriculum, in particular 1) that canons may be written at different pitch and time intervals; 2) that they may unfold in accordance with other, more complex "rules"—e.g., retrograde, inversion, augmentation, and diminution; and 3) that they are not simply pieces that exist in sight-singing manuals designed to make undergraduate music majors learn solfège. Some subsidiary benefits of the exercises include introducing students to the *Goldberg Variations* as a piece of art music with its own history and afterlife and familiarizing them with da capo variations form, the Sarabande as a stately triple-time Baroque dance, the quodlibet as an eighteenth-century "mashup,"[2] and Bach's highly rigorous, contrapuntal manner of composition.

Each of the three exercises on Bach's canons (as well as a fourth, non-canonical one) fits on two pages. I typically print them on the front and back of one leaf and present them more or less as is. But I am also including clean PDFs of the templates, in case instructors wish to present them along with other introductory material. The exercises are:

1. Listening and Improvisation: The Canon at the Sixth (p. 3)
2. Discussion and Demonstration: The Canon at the Second (p. 5)
3. Analysis and Completion: The Canon at the Fourth (p. 7)
4. A non-canonical figured-bass realization exercise[3] (p. 9)

To keep them to a manageable size, all four of the exercises involve only the first reprise of the relevant variation, although, if there is interest, instructors can easily build in time to look at their second reprises. Because the first reprises all modulate (to the dominant), the exercises are most appropriate for students who have been exposed to modulation.

The next page provides a basic introduction to the *Goldberg Variations* as I would present it in class. The worksheets follow.

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[1] Such exercises may be found, e.g., in William Renwick, *The Langloz Manuscript: Fugal Improvisation through Figured Bass* (Oxford University Press, 2001) and David Ledbetter, *Continuo Playing According to Handel: His Figured Bass Exercises* (Oxford University Press, 1990).

[2] During the 1955 recording sessions of the *Goldberg Variations*, Glenn Gould gave a charming introduction of quodlibet to one of the sound engineers, using The Star-Spangled Banner and God Save the Queen as themes. The quodlibet was made available on a disc of outtakes from the recording sessions called "A State of Wonder" and may be found here: https://www.youtube.com/watch?v=FLdKGIzkKT0.

[3] An idea of how artistic and intellectually robust such figured-bass realization exercises may be and how central they were to Bach's own pedagogy is given in Vasili Byros's "Prelude on a Partimento," *Music Theory Online* 21.3 (2015).

Introduction

I present the *Goldberg Variations* to my class as an essay in canon writing. I emphasize that (the cerebral, "recherché, arcane, speculative") Bach set himself a compositional challenge: to write thirty variations on a bass line, the so-called "Goldberg ground," of which every third variation is a canon at a different diatonic interval.[4] The contrapuntal fun begins with Variation 3, which is a canon at the unison, and continues until Variation 27, which is a canon at the ninth.[5] With one exception (Variation 27), all of the canonic variations involve two strictly canonical voices and a third accompanimental voice.

An idea about Bach's compositional prowess can be gleaned from the following story, which I typically tell my students as a setting of the stage. In 1974, a musicologist named Olivier Alain discovered Bach's personal copy (or *Handexemplar*) of the published version of the *Goldberg Variations* in Strasbourg, France.[6] At the end of the *Handexemplar*, Bach wrote fourteen (additional) canons on the first eight notes of the *Goldberg* Ground, in order of increasing complexity.[7] (The number 14 is significant for Bach: if you convert the letters of the alphabet into numbers such that A = 1, B = 2, and so on, 14 is the sum of B, A, C, and H.)

This YouTube video from a user named Gerubach gives an introduction to the discovery of the *Handexemplar* and also provides animations of each of the canons. The video can be exceedingly fun to watch in the classroom; I have seen it stimulate both interest and awe. I often skip around in the video to show a couple of the more impressive canons, and I always show the last one, with its four levels of diminution. The canons may be too complex to teach formally in a third-semester theory classroom, but the videos make apparent their contrapuntal rigor.

It is significant, I think, and worth pointing out after watching the animation of the last canon, that after it, Bach wrote the abbreviation "etc." According to Christoph Wolff, this "shows clearly that the composer could have continued with further canons had he wanted to."[8]

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[4] The adjectives in parentheses are from David Yearsley, *Bach and the Meanings of Counterpoint* (Cambridge 2002), xiii. The first eight notes of the subject, or *soggetto*, of the Goldberg Ground was not written by Bach; it dates back to the early seventeenth century.

[5] Var. 30 is not a canon but a "Quodlibet"—a mash-up of two popular songs that is also constructed in such a way as to preserve the Goldberg Ground. See again n. 2.

[6] See Christoph Wolff, "Bach's *Handexemplar* of the Goldberg Variations: A New Source," *Journal of the American Musicological Society* 29/2 (1976), 224–241.

[7] The additional canons have their own BWV number, 1087. Students who are beguiled by Bach's solutions to the "problems" that he sets himself in these miraculous canons may be interested in Matthew A. Haakenson's "Probing the Properties of the First Eight Notes of Bach's Goldberg Ground Further: Revelations for Replicating the *Soggetto* and Opening Four Canons of BWV 1087," *Music Theory Spectrum* 39/2 (2017): 247–260. Instructors might also glance at the language used to discuss the "alchemy of Bach's canons" in Yearsley, op. cit.

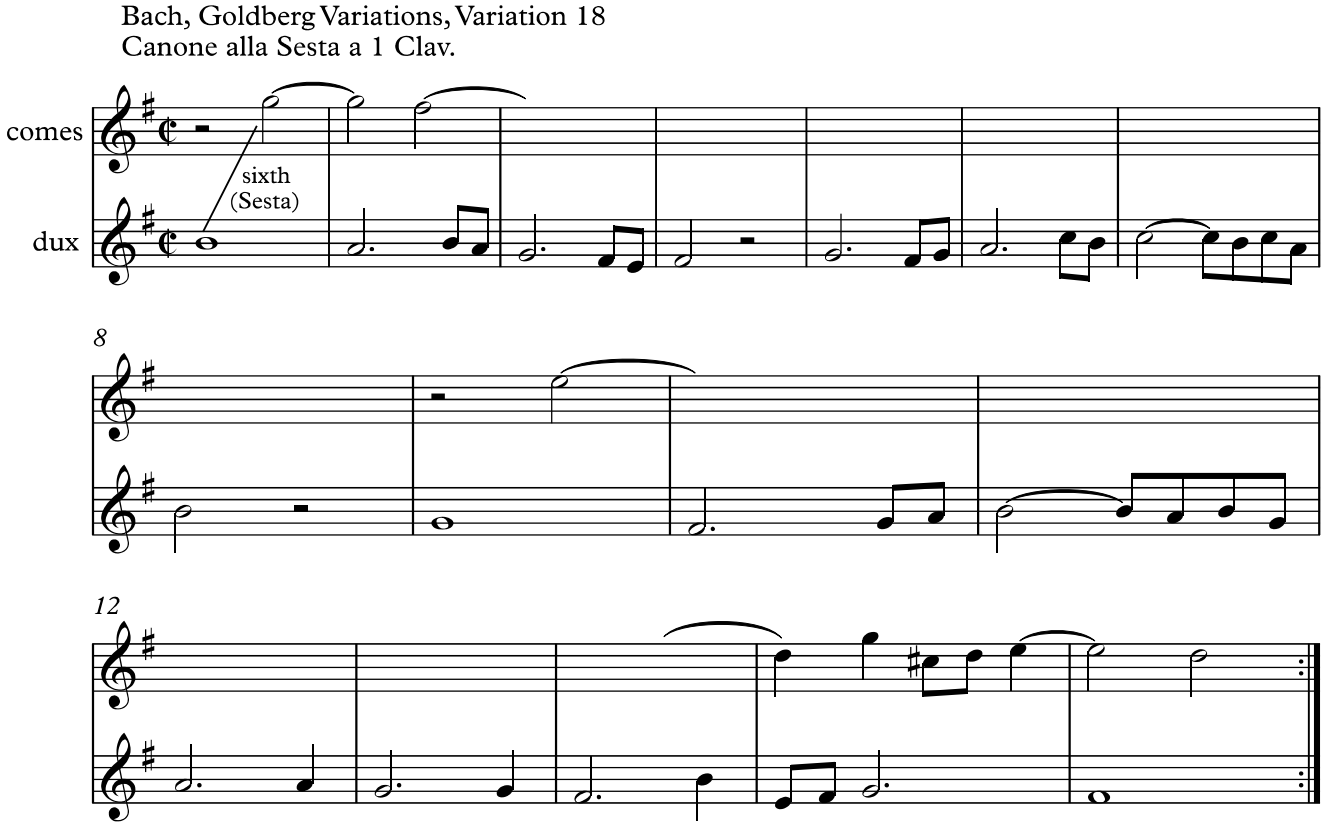
[8] Wolff, "Bach's *Handexemplar*," 231.

I. Listening and Improvisation Exercise: Var. 18. The Canon at the Sixth

In this exercise, you will improvise the "following voice" (*comes*) of a canon by Bach in real time. To do so, you will have to listen very closely to the "leading voice" (the *dux*) and, at the same time, stay focused on their line, which is always a sixth above the *dux* and a half note behind. We proceed in four stages.

Stage One:
All students sing the *dux* (the "leading voice") only of this canon while I play Bach's accompaniment. The *dux* is given on the bottom system of the example, below. The accompaniment is given on the score on p. 2.

Stage Two:
Half of the students sing the *dux* again, while the other half improvise the *comes* (the "following voice") in real time. The *comes* is a sixth above the *dux* and a half note behind. Where needed, I will give some support from the piano. After a successful performance, we will switch roles. Some of the *comes* pitches are given.



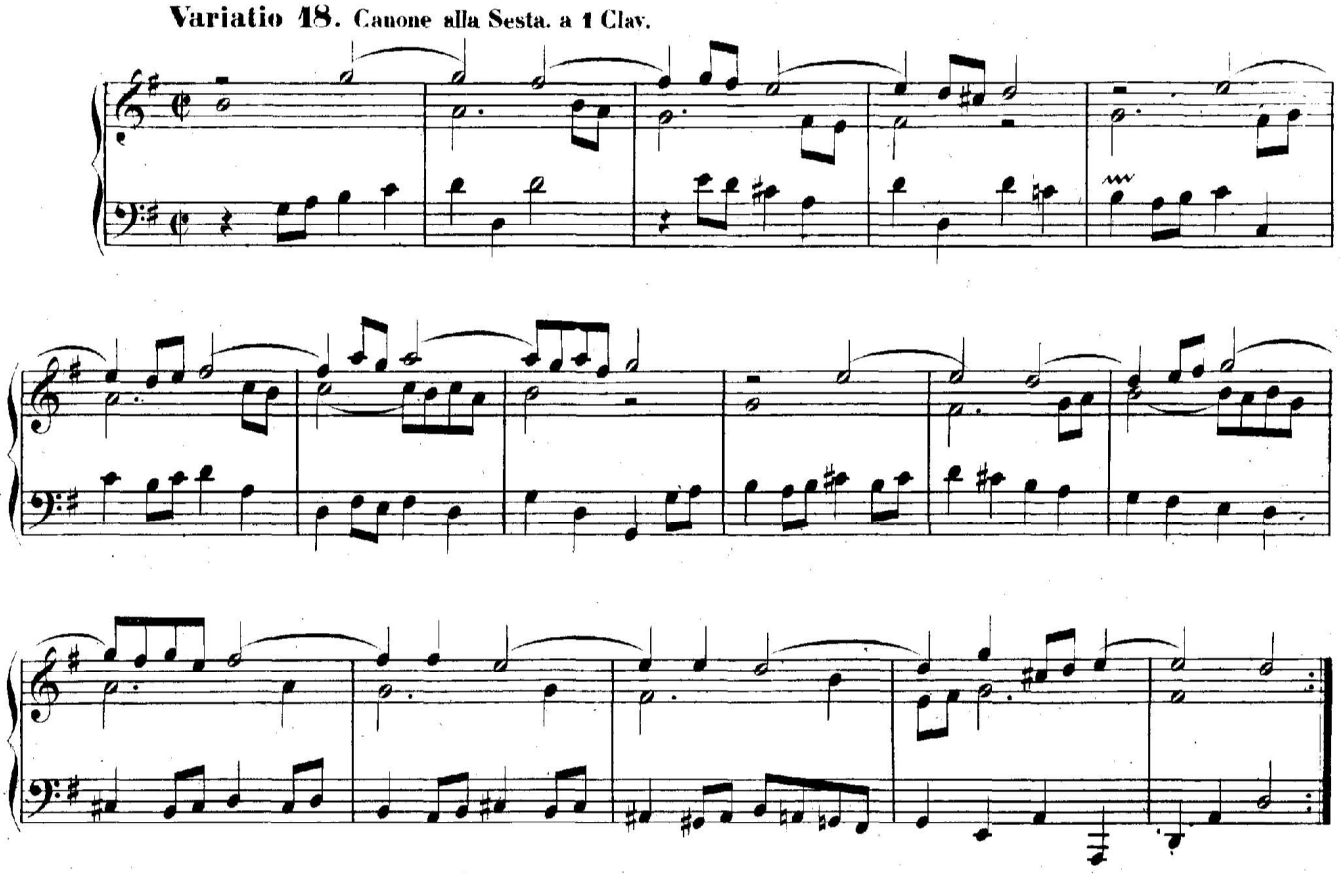
Stage Three (optional):
Write in the rest of the pitches for the *comes* voice.

- NB: Use a decorative C♯ (not a C natural) in m. 4; everything else is strict

Stage Four:

Sing both voices from notation (on the back of this handout), while I play Bach's accompaniment.
Analyze with Roman numerals.
Score for Analysis

From the Bach-Gesellschaft Edition, Leipzig, 1863



DOI: 10.71156/2994-7073.1380

II. Discussion and Demonstration: Var. 6. The Canon at the Second

After doing the improvisation exercise this year, one of my students commented that a canon at the sixth couldn't be *that hard* to write, since the sixth is a consonant interval. But the protest is not entirely apropos, since with canons one is not talking simply about a vertical interval (a harmonic sixth, say), but rather about two moving lines. That the second is a copy of the first—say, "at the sixth"—does not mean that the two lines combine to form sixths at every time point. So it is not so much whether the organizing, canonical interval is consonant or dissonant but how a *dux* may be constructed in such a way that it brings about a satisfactory *comes*.

A clever example of a canon at a "dissonant" pitch interval may be found in Var. 6 of the *Goldberg*s, the canon at the second. Here it becomes clear that the problem is not so much what to do with the dissonant (vertical) interval of a second but rather to figure out how to proceed when every pitch the *dux* sings will produce a *comes* pitch a step above it *in the following measure*.

There are several beautiful touches here that show Bach's solutions to the "problem" of the second. Four of them are described here and highlighted on the following graphic.

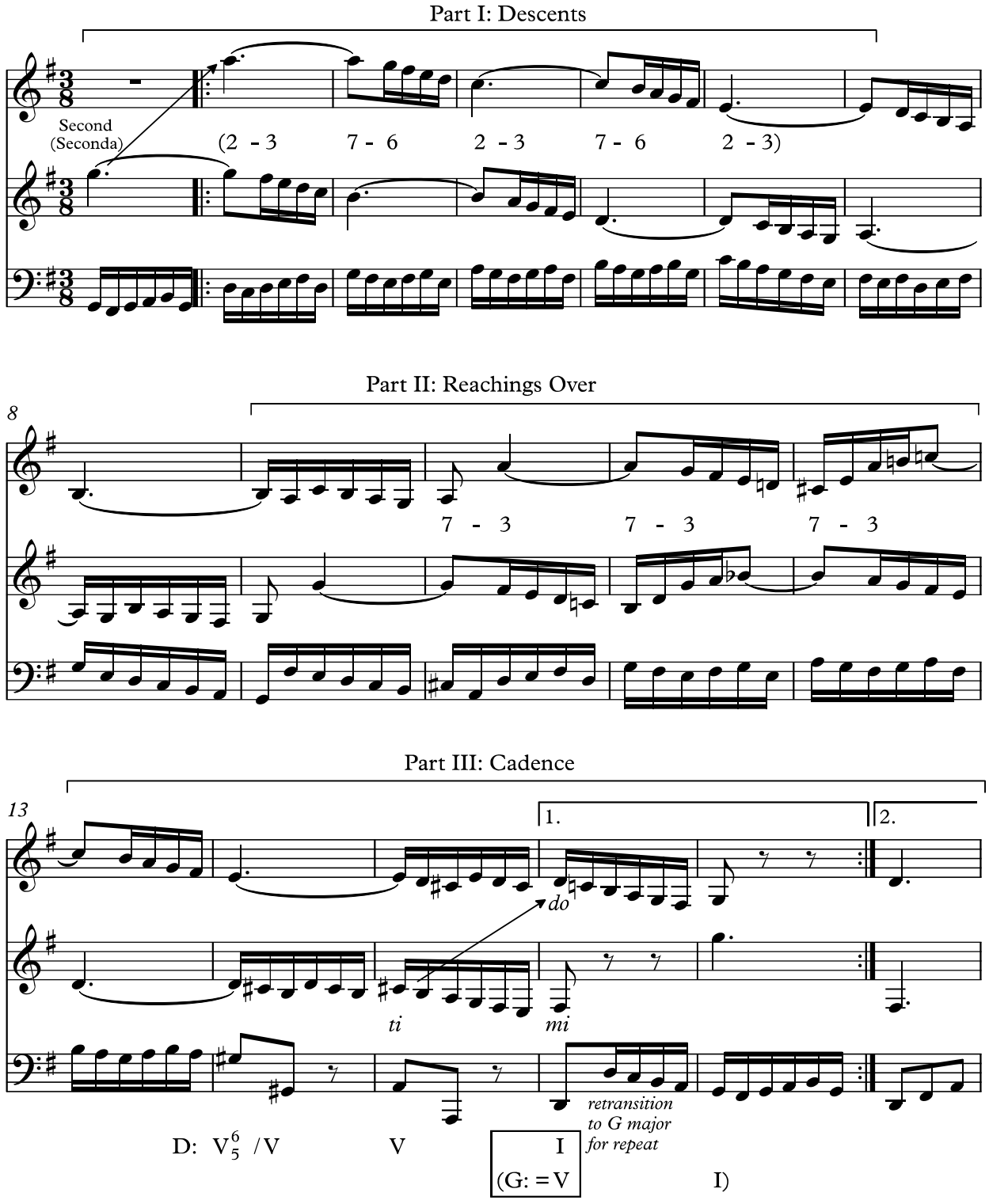
1. On the first system (call this Part I of the reprise), the long note values in the *dux* voice lead to a dissonant suspension chain (2-3, 7-6 …) in the top two voices. These are labeled between the staves.

2. In Part I, the canonical voices only descend, which means that the *dux* voice is always below the *comes*. In Part II, however, on the second system, Bach begins a pattern of reaching over, such that the *dux* and the *comes* begin to intermingle in the same register, overlapping one another. (Note the ascending stepwise line that emerges from this game of leapfrog: G5 in m. 9, A5 in m. 10, B♭5 in m. 11, C6 in m. 12.) This has implications for the suspension types: are they still 2-3s? Are they 7-6s? 7-3s?

3. Particularly beautiful is the way that Bach makes his final cadence (in D Major). As shown on the annotated score below, at m. 15, the C♯ of the *dux* is the leading tone in a V chord in the key of the dominant, D major. Knowing well that the C♯ *dux* on the downbeat of m. 15 will result in a D on the downbeat of m. 16 in the *comes* voice, Bach lets the leading tone in the *dux* voice move *downwards*, tucking in to an inner voice tone F♯ to provide the chordal third. This C♯-to-D motion, across voices, is a particularly clever solution to the cadence-problem that comes with writing a canon at the second.

4. Note that the melodic motion just described, from C♯ down to F♯ in m. 15–16 (from scale degree 7 *down* to scale degree 3—what I called a "tucking-in" to an inner voice) makes for a retransition in the *comes* voice that moves from the D-Major PAC at m. 16 back to G major for the repetition of the reprise.[9]

[9] The final cadence of the first reprise in the canon at the ninth (Var. 27) is similar and provides a nice comparison. But note: the opening of the second reprise in Var. 27 begins on A and is presented in inversion. The *comes* voice then enters a ninth below it. This makes possible an immediate return from V to I, to accommodate the Goldberg Ground. By contrast, in Var. 6, the subject of the canon appears right side up after the double bar, and the interval is still an ascending second.



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III. Canon Completion Exercise: Var. 12. The Canon at the Fourth.

On the back of this paper, you will find the entire *dux* voice as well as the non-canonical accompanimental voice for the first reprise of the *Goldberg* canon at the fourth. Complete the reprise by adding the *comes* voice, which begins in m. 2 at the time distance of one dotted quarter note and (beginning, at least) at the interval of a fourth below. Then play your solution. Some pitches are given to you.

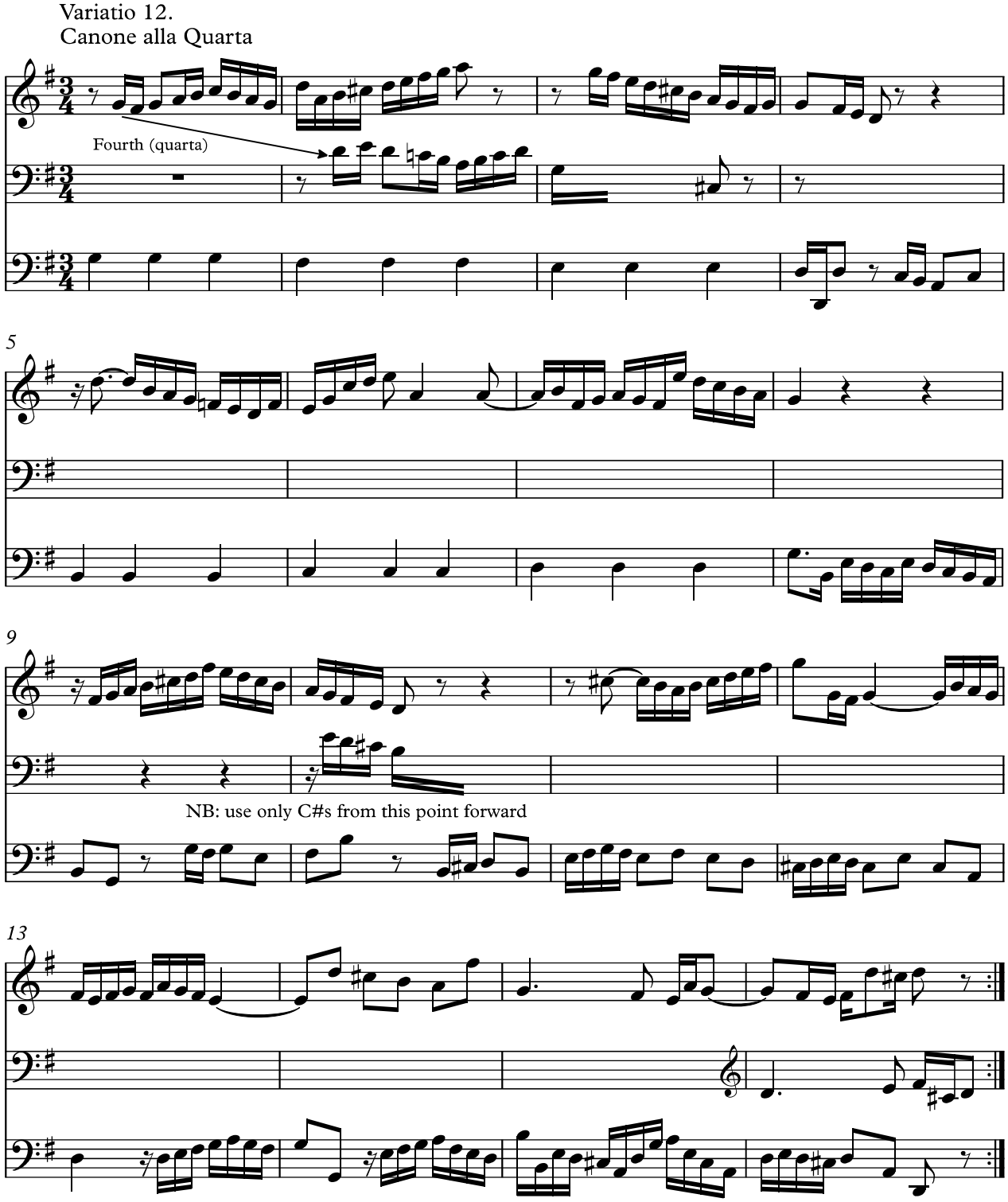
Important! What "rule" has Bach used to write the *comes* voice of the canon? What effect does this have on the generative interval of the canon? This may be made clearer by completing the chart below.

- You may change the clef where convenient
- Use only C♯s in the *comes* after the modulation occurs. I have provided the first *comes* C♯ for you at m. 10.

Canon at the "Fourth": Chart of Intervals

Bonus: What happens at the dotted line?

WHEN THE *DUX* SINGS THE PITCH	IT YIELDS THE *COMES* PITCH	THE INTERVAL IS
C4		
D4	G4	4th
E4	F♯4	2nd
F♯4	E4	2nd
G4	**D4**	**4th**
A4	C4	6th
B4	B3	8ve
C5		
D5		
E5		
F♯5		
G5		
A5		

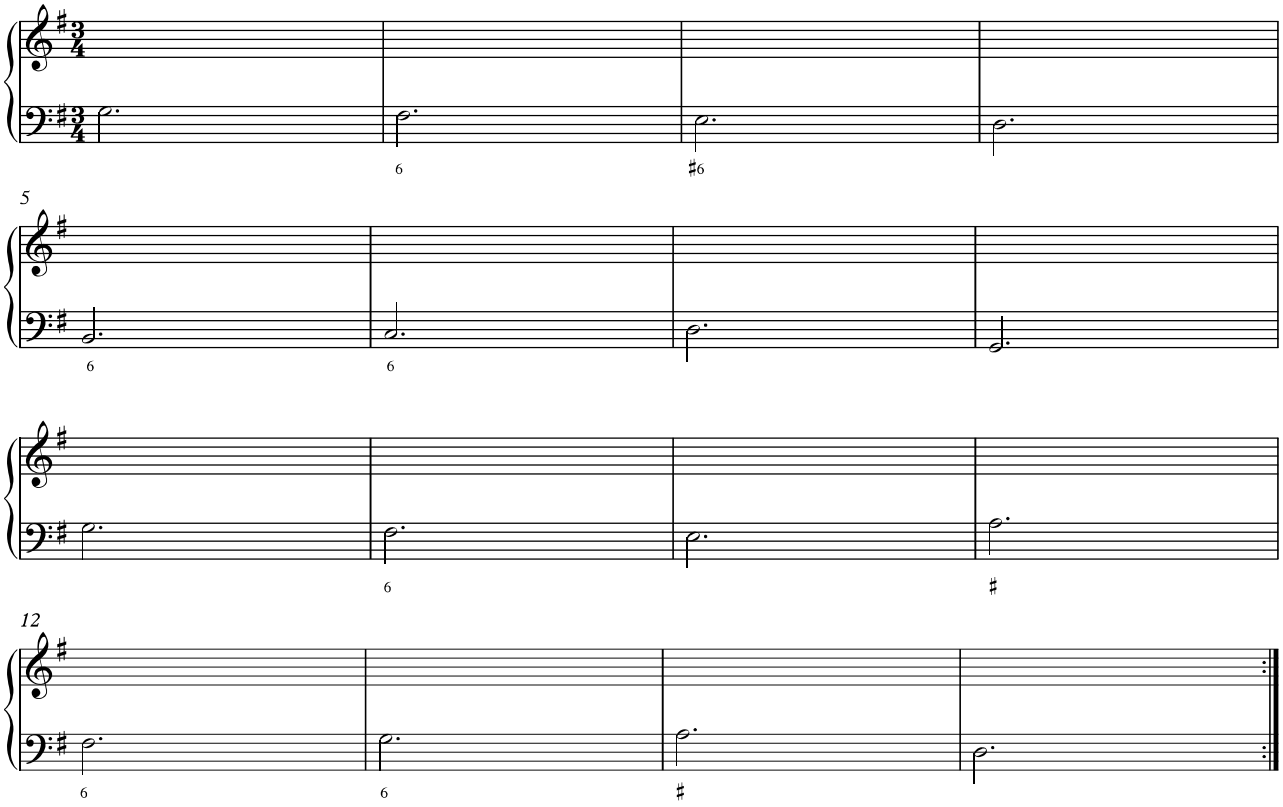

Variatio 12.
Canone alla Quarta
Fourth (quarta)
5
9
NB: use only C#s from this point forward
13

IV. Figured Bass Realization Exercise: The Goldberg Ground.

Analyze and realize the first reprise of the Goldberg Ground in three or four obbligato voices. (You may switch between three and four voices; Bach does so.)

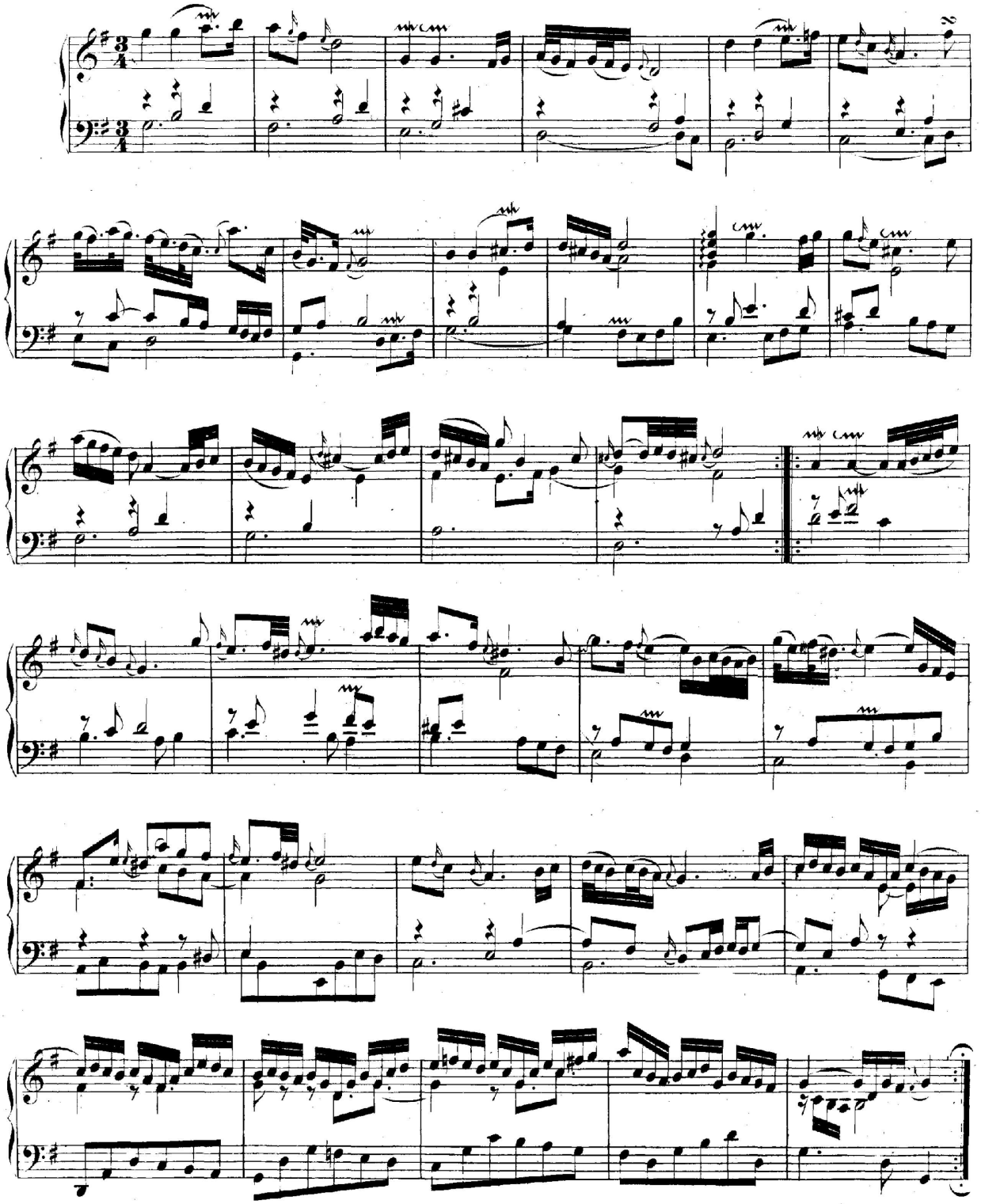
Strive for rhythmic and melodic independence of all of your lines and for a singing upper voice. Bach's Aria is a stately Sarabande. Experiment with ornaments. Label chords with Roman numerals and label modulations. Be sure to make a PACs in the final measures of the reprise.

When you have finished, compare Bach's realization of the Ground as the Aria of the *Goldberg Variations*, given in its entirety on the next page.[10]



[10] For a beautiful example of a 21st-century realization of an 18th-century figured-bass exercise, see again Byros's "Prelude on a Partimento," accessible at http://www.mtosmt.org/issues/mto.15.21.3/mto.15.21.3.byros.html

ARIA.



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DOI: 10.71156/2994-7073.1380