The Analysis and Composition of Polymorphic Canons

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Although theorists have examined various aspects of the canonic genre, scant attention has been given to the polymorphic canon, a designation which Prout bestowed on those pieces with multiple solutions. This article focuses on two prototypical examples, the anonymous *Non nobis Domine* and a seven-measure *dux* by G. H. Stölzel quoted in Marpurg's *Abhandlung*, from whose analyses we may extract recurring melodic paradigms prior to providing practical guidelines for their composition.

Matthew Haakenson's insightful discussion of the canons which Bach sketched inside the cover of his personal copy of the *Goldberg Variations* continues the recent interest in this intriguing musical genre, as evidenced by articles of Robert Morris, Robert Gauldin, and Alan Gosman.[1] Commencing with such Renaissance notables as Glarean, Zarlino, and Morley and working through Berardi, Albrechtsberger, and Marpurg to Ebenezer Prout in the late nineteenth century, theorists have examined numerous aspects of this strict discipline, such as canons employing inversion, retrogression, diminution and/or augmentation, as well as examples of double, stretto, infinite, modulating, accompanied, and puzzle canons.[2] However, scant attention has been paid the *polymorphic canon*, a designation bestowed by Prout on those pieces whose dux may be realized in more than one way, or in other words, canons with *multiple* solutions.[3]

Most canonic commentaries are limited to providing examples from the literature, offering solutions to puzzle pieces, or supplying general observations on formal/tonal structure. Only rarely do they specify compositional problems inherent in particular

1 Haakenson (2017); Morris (1995) establishes a series of rigorous and exhaustive paradigms for two-voice canons in various styles; Gauldin (1996) proposes algorithms for the composition of three-voice symmetrical/asymmetrical stretto canons in 16th-century style; Gosman (1997) discusses the characteristics of "stacked" canons, whose entries occur on different pitch classes, such as successive perfect 4ths or 5ths; and Gosman (2009) isolates melodic-harmonic patterns of first-species reductions and their application within a wide range of the subsequent
two-voice canonic literature.

2 Bach's "canonic variations" on Frederick's "Royal Theme" in his *Musical Offering* display no fewer than five of these varieties.

3 See Prout (1891, 249–253).

canonic types or the manner in which these issues have been resolved. Nor do they impart much in the way of practical guidelines for the prospective composer, all of which imply more pedagogical approaches. The following discussion focuses on two celebrated pieces that will serve as prototypical examples, from whose analyses we may extract recurring melodic paradigms prior to providing practical guidelines for their composition: the anonymous *Non nobis Domine*,[4] and an untitled piece by G. H. Stölzel,[5] quoted in Marpurg's *Abhandlung von der Fuge* ([1753–54] 1970).[6] Both are exemplars of *stretto* canons, in that their initial point of imitation is complete within the first few measures, a rather common procedure in the Renaissance.[7] Although this pair share certain characteristics, they nevertheless represent different ways in which the *dux* may be utilized in the imitative process.

The analyses will incorporate first-species reductions, previously exploited by Morris, Gauldin, and Gosman as a means for examining their underlying structure. Example 1 lays out the *dux* of *Non nobis Domine* with its Latin text based on Psalm 113, verse 9, as well as its semibreve reduction aligned below.[8] The surface melodic elaborations of this first-species background will prove crucial in its subsequent realizations. The durational alterations in mm. 2 and 6 permit a continuous rhythmic flow between the various voices. While the inserted B_4[9] in m. 1 results in simple passing motion, the remaining pitch changes in mm. 4 and 5 either prevent parallel/direct motions to perfect intervals or produce various types of suspensions.

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4 The *dux* of this notable Renaissance canon was supposedly compiled by an unknown English composer from two melodic phrases of a motet *Aspice Domine* by Philip van Wilder (d. 1554), although as the subsequent reduction of mm. 1--3 and 4--6 reveals, their underlying first-species frameworks are identical. Several realizations of this piece, previously attributed to William Byrd, may be found under "Canon" in *The New Grove Dictionary of Music and Musicians*, 2nd edition (2001), Stanley Sadie, editor.

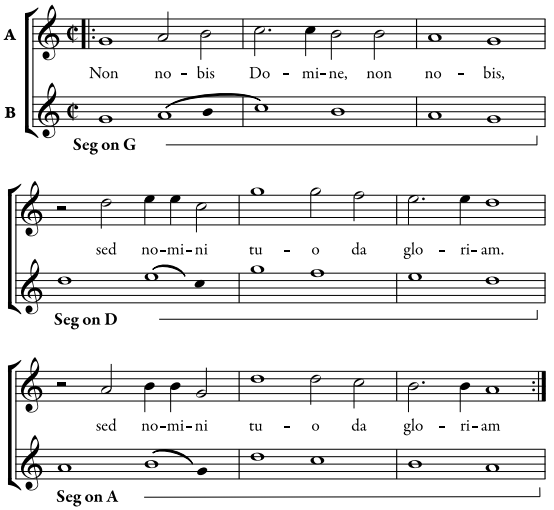
5 Gottfried H. Stölzel (1690–1749) was a German composer of numerous operas, oratorios, and church cantatas.

6 Marpurg's remarks on canon ("Vom Canon") comprise Chapter 6 of Volume II.

7 This process contrasts with those lengthier and more self-contained fugue subjects of the Baroque period.

8 For his meticulous preparation of the musical examples for this article, I would like to thank Richard Desinord, a Ph.D. candidate in theory at the Eastman School of Music.

9 The system of pitch designation employs that of the American Acoustical Society, where middle C = C_4 and an octave higher = C_5.



Example 1
The *dux* of *Non nobis Domine* and its first-species reduction.

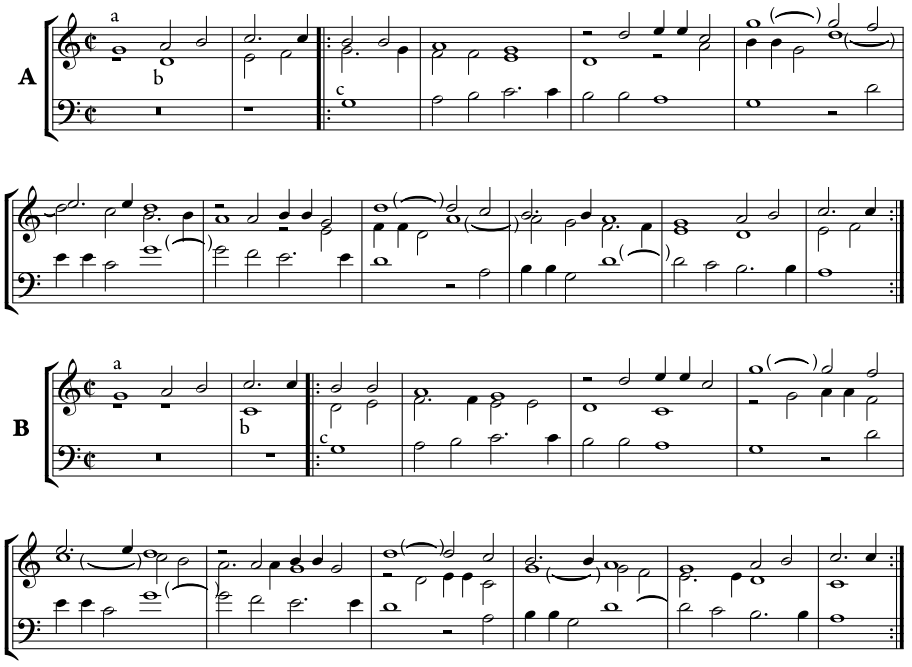
The above reduction reveals a basic six-note melodic *segment* (pcs G A C B A G) whose two successive sequences ascend by fifths—first on D and then on A (denoted by brackets). This canonic technique, hereafter designated as *segmental sequence*, is discussed in Gauldin (1996), which cites several examples.[10] Sequential extensions of this initial six-tone segment can produce *perpetual* or *infinite* canons of various lengths.[11] For instance, the second sequence (on A) may be succeeded by a restatement of the original segment (on G), as indicated by the inserted repeat signs in Example 1. These sequences may also continue as a series of ascending fifths which cycle through the entire diatonic collection.

Analytical commentary on these polymorphic canons will incorporate the following abbreviations. The individual tones of the different voices (*a*, *b*, *c*, etc.) are indicated by superscripts (a^1, a^2 , a^3, a^4), melodic motion (MI) by plus or minor diatonic intervals (+2, -5), diatonic harmonic intervals between voices (HI) in terms of their relationship to the bass (8/3, 5/3, 8/6) or to each other, the interval of imitation between successive entries by P, and the temporal distance between entries in semibreves by T. The pair of perpetual three-voice realizations in Examples 2a and 2b exemplify *stretto* canons which employ *asymmetrical* tonal and temporal distances

10 See especially the examples by Josquin and Palestrina in Gauldin (1996, 31 and 36).

11 As we will see later, this option is not possible in four-voice solutions.

between entries, such as P-4/P-5 tonally or T1/T2 temporally. Calculated from the initial upper voice, Example 2a employs a *dominant*-oriented relation (P-4/P-5), whereas Example 2b features a *plagal*-oriented relation (P-5/P-4), both characteristic of Renaissance imitative technique. Although some elaborative dissonance appears in the form of 7–6 and 2–3 suspensions, the ascending accented passing tone A_4 in both solutions runs counter to accepted stylistic practice.



Example 2
Two perpetual three-voice settings of *Non nobis Domine*.

Since by definition, a polymorphic canon is capable of multiple solutions, it is possible for a trio of distinct *biciniums*, each with differing imitative characteristics, to reside within a single three-voice realization, provided no dissonant harmonic intervals occur between the voices. Therefore, the three embedded two-part canons in Example 2a may be indicated as *a* → *b* (P -4/T1), *b* → *c* (P -5/T2), and *a* → *c* (P -8/T3). A comparison with Example 2b reveals that although b involves different pcs, it exhibits no "new" embedded canons, since the indicators remain constant: *a* → *b* (P -5/T2), *b* → *c* (P -4/T1), and the original *a* → *c* (P -8/T3). Disregarding "redundant" canons

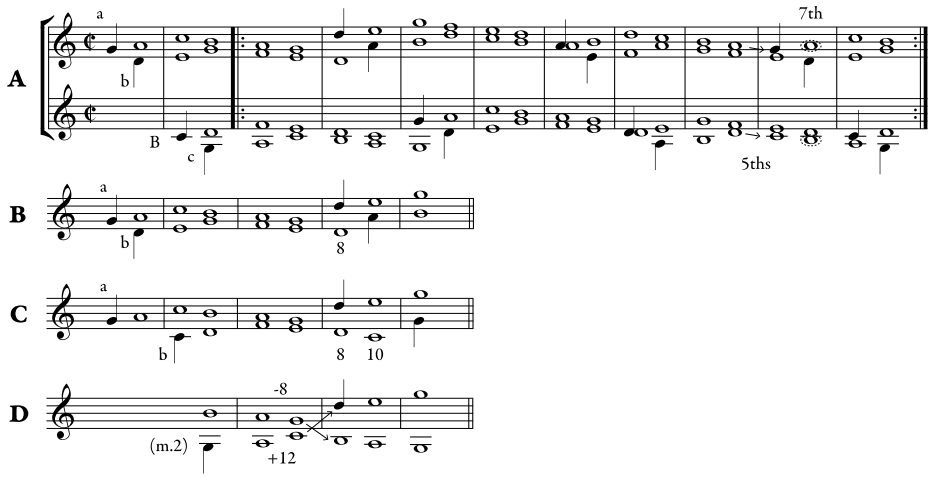
whose basic indicators form duplicates, there are seven different canons embedded within the possible three- and four-voice solutions of this particular *dux*, assuming we remain within the dominant/plagal imitative paradigms.

The following analytical commentary will focus on three specific sections of the *Non nobis Domine* canons, employing first-species reductions: 1. the initial point of imitation, 2. the commencement of the segmental sequences, and 3. the concluding voice overlap necessary to produce a perpetual canon.

The initial point of imitation. It requires only a few cursory notes to realize that (P -5, -4/T1, 2) and (P -4, -5/T2, 1) are not possible.[12] An expedient procedure for examining its two remaining realizations (P -4, -5/T1, 2) and ((P -5, -4/T2, 1) would be to construct a hypothetical "four-part" solution including both dominant and plagal imitation, as shown in the first-species reduction of Example 3a. Each black note-head and stem denotes the beginning of a six-note segment. The interior voices (commencing on D_4 and C_4) are denoted as *b* and *B* respectively, in which *b* is notated as the "alto" voice and *B* as the "tenor" voice. In working out the initial point of imitation (mm. 1–3), the basic problem involves the two interior voices (*b* and *B*), which lie only a 2nd and one semibreve apart. To counter this quandary, the composer of this canon devised an ingenious solution. If the *dux* initiates a *descending stepwise* motion from C_5 down to G_4 to complete the original segment, then three of the pcs in *b* and *B* (F_4, E_4, and D_4) will *coincide*. As a result, it is only necessary to compute mm. 3 and 4 in terms of *three* voices. Consequently, the original *dux* segment is *framed* by a motion from $G_4 \rightarrow A_4$ and $A_4 \rightarrow G_4$, a feature which will prove crucial in the subsequent segmental overlaps.

The segmental sequences. Two sequential restatements by ascending fifths succeed the original dux segment, commencing with D_5 and A_4 in mm. 4 and 7 of Example 3a, respectively. In the process, the initial pitches of its first sequence must *overlap* with the lower voices *b*, *B*, and *c* by one, two, and three semibreves, respectively, a potential problem which the unknown composer cleverly alleviates. Since the first *a* segment commences with $G_4 \rightarrow A_4$ and concludes with a regressive $A_4 \rightarrow G_4$, the last tone of the dominant-oriented *b* segment (D_4) forms an octave with the initial D_5 of the *dux's* first sequence in m. 4 (Example 3b). In a similar manner, the last notes of the plagal-oriented *B* segment ($D_4 \rightarrow C_4$) form an consonant 8va and 10th with the $D_5 \rightarrow E_5$ of the *dux's* first sequence in m. 4 (Example 3c). Finally, because the

12 Given the initial pitch, the point of imitation indicators, and successive melodic intervals (MI) of the *dux*, the pcs of the harmonic sonorities generated in three-voice settings may be ascertained using mod7 algorithms, although their complexity renders such calculations tedious.



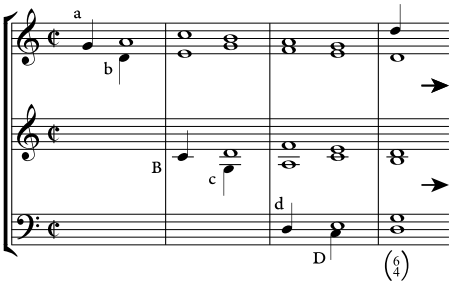
Example 3
A hypothetical "four-voice" realization of the *dux* in reduction.

original *dux* segment is *six* semibreves in length, and the *c* voice (an octave lower) does not enter in *both* cases until *after* three semi-breves, it is possible to *exchange* their trichords by employing double counterpoint at the 12th, in which the last three pcs of the original *a* segment descend an 8va lower to become the last three pcs of the *c* segment, while the first three pcs of the *c* segment ascend a twelfth to become the first three pcs of the second *a* segment (consult Example 3d).

The "perpetual" overlap. One final hurdle remains in our perpetual realization, since at some point the *dux* must "cycle back" on itself. This restatement of the original segment on G_4 must substitute for one of the sequential segments, creating an overlap of the concluding pcs in *b*, *B*, and *c* of the previous segment. In such a situation, does one particular positioning of this reprise prove more opportune than another? If its recurrence of the original *dux* segment substitutes for the anticipated segmental sequence on E_5, then no harmonic problems arise with overlaps of the *middle* voices *b* and *B* (consult the final measures of Example 3a). However, our reduction does reveal two potential conflicts with the bass or *c* voice. Not only do parallel 5ths occur, but also the harmonic interval of a seventh appears between mm. 9–10. While the subsequent elaborations (refer to Example 2) cleverly disguise the former by means of a bass suspension, the rationalization of the *dux* A_4 as "passing motion" is less convincing, since such melodic dissonance was traditionally limited to *descending*

minims on *weak* beats 2 or 4.

The possibility of expanding dominant/plagal realizations for *Non nobis Domine* to four-voice texture considerably raises the ante; consult the "dual" reduction in Example 4. The former interior voices *b* or *B* now reappear as the *d* or *D* voices to assume the *bass* function previously performed by *c*. While the imitative entries between *a*, *b*, and *B* are duplicated an octave lower as *c*, *d*, and *D*, and the previous relations between *a*, *b*, and *c* recur a 4th lower as *b*, *c*, and *d*, the problem remains that the opening pcs of *d* and *D* must overlap with the last pcs of *a*, *b*, and *B* during the original *dux* segment; consult the "dual" first-species realization in Example 4. No harmonic conflicts occur within the initial point of imitation, but as the piece progresses, one encounters increasing instances of harmonic sevenths and parallel octaves/fifths.[13]



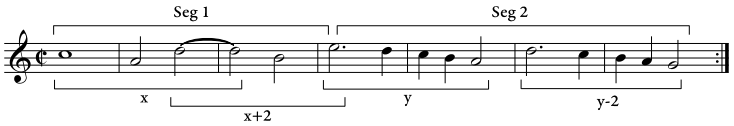
Example 4
A "dual" four-voice realization of *Non nobis Domine's* point of imitation.

While it must be admitted that some of the solutions of *Non nobis Domine* are flawed, no such aberrations appear in Sötlzel's polymorphic canon, which (in the words of the Emerald City's gatekeeper) is a "horse of a different color." In the former piece, the treatment of dissonance (passing tones and suspensions), the absence of motivic sequences, and non-functional harmonic progressions typify normative Renaissance practice. However, the latter work clearly represents a product of the subsequent tonal period, as characterized by its brief sequential patterns, block-chordal construct, and directed harmonic motion. Of its numerous realizations,[14] the subsequent discussion will consider only three, each of which exhibits its own distinctive features. Whereas

13 The fourth between *d* and *D* and the parallel fifths between *B* and *d* in m. 4 will disappear when the three realizations appear separately.

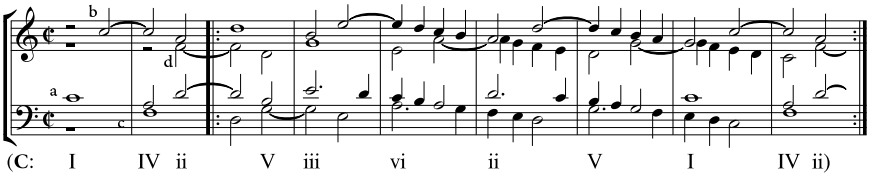
14 Marpurg calculates that there are 392 possible realizations, including inversion and *arsin et thesin*.

Non nobis Domine involved a *single* six-note segment whose successive statements rose by cyclical fifths, a cursory glance at Stölzel's *dux* reveals a pair of seemingly contrasting segments (bracketed and denoted as Seg1 and Seg2 in Example 5), each of which consists of a three-note melodic cell (labeled Motive x and Motive y) that is immediately sequenced; while the first ascends stepwise, the second descends by step. The concluding pitch in Seg1 (the -3+4 motive) overlaps with its next statement, but no such imbrication occurs with the -3-3+4 motive in Seg2 (note the embedded -3+4 cell). All imitative distances are calculated in terms of half-note durations.



Example 5
Dual segmentation of the *dux* in Stölzel's canons.

In the first realization (hereafter labeled "Stolzel 1" and shown in Example 6),[15] the *dux* is immediately subjected to a plagal-oriented four-voice stretto (P+8,-12,+8) whose entries occur at a distance of one half-note (T1,1,1). The two segments not only elide midway through but overlap in various ways during the closing measures to produce a perpetual canon.

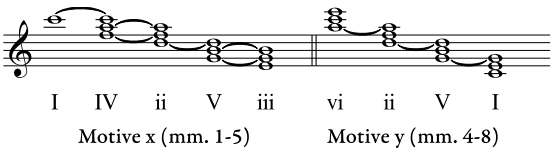


Example 6
"Stölzel 1" realization.

A comparison of Motives x and y in these stretti reveals their close relation. Both originate from a basic descending MI -3, whose octave imitation occurs one minim later. On the other hand, their subsequent sequences take divergent directions—Motive x moves upward by 2nds (outlining I–ii–iii triads) while Motive y descends stepwise through a harmonic cycle of fifths (vi–ii–V triads) back toward tonic; an E-minor sonority functions as the connective link between the two stretti. The fifth cycle in Seg2 culminates in I–IV (mm. 8–9), which allows an effortless overlap back to the

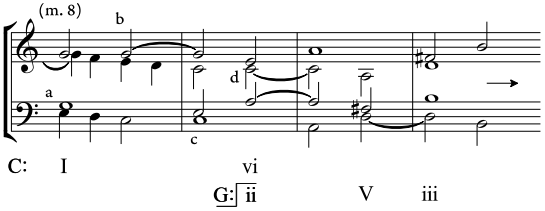
15 Actually, this realization represents a later version; the original canon is shown as "Stölzel 3."

canon's opening progression. But how is Stölzel able to facilitate these stretti with such apparent ease? The key to his success appears to lie in the retention of *common* tones between successive harmonic sonorities. Whereas the pcs in the chordal progression of the initial Seg1 projects a downward chain of 3rds, the following Seg2 reverts to the time-honored technique of falling fifth relations, revealing another third chain (consult Example 7).



Example 7
Chains of thirds in the harmonic progressions of "Stölzel 1."

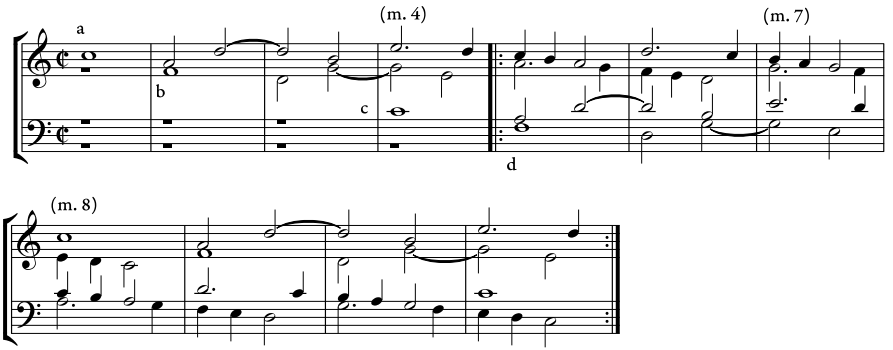
The conversion of this piece into a modulating canon (= "Stölzel 2") that would eventually cycle through the *entire* chromatic spectrum of keys by ascending fifths presents few problems. Rather than reverting to the original C which jump-starts our former perpetual version, the concluding overlap now utilizes the pcs of the *dominant* function (G B D) to initiate a point of imitation in G major (as shown in Example 8), a process that is perpetuated throughout the remainder of the canon.



Example 8
The pc overlap that commences the cyclical modulations in "Stölzel 2."

Although our final realization ("Stölzel 3" in Example 9) retains the initial two-voice plagal stretto (P -5/T2, albeit displaced an octave), it displays a different set of characteristics. In some ways it even resembles a *round*, in which a series of distinctive melodic phrases are successively accumulated and contrapuntally aligned prior to their perpetual repetition. Since in this case, only two "phrases" (or segments) are involved, one might suggest the term *segmental superimposition*. A vertical alignment

of Seg1 and Seg2 takes place in m. 4, followed by octave exchange in mm. 7–8, although their simultaneous combination must also take into account the accompanying two-voice stretto of each. Since Stölzel's canon is *seven* (rather than eight) measures in length, resulting in a "misalignment" of the segments, a simple double-counterpoint switch at the octave will not suffice (compare mm. 7–8 with m. 4). On the other hand, the concluding pc overlaps which are necessary for a perpetual canon are effortlessly realized, as the final measures of Example 9 illustrate.



Example 9
The realization of "Stölzel 3."

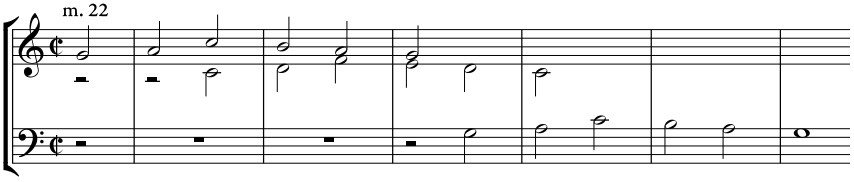
Of other existent polymorphic canons which could supplement our previous analyses, we will cite only two.[16] Marpurg quotes an example by Valentini which he claims is capable of over 2000 different realizations. But perhaps the most frequently cited instance is the two-voice Canon No. 10 in the *Musical Offering* which exploits inversion, and over which Bach inserted the cryptic scriptural inscription *Quaerendo invenietis* ("Seek and ye shall find"). It features prefatory clef signs for its proper realization (an upside-down bass clef), but no indication as to when the second voice enters. Although no fewer than *four* "solutions" of this puzzle canon appear in the *Bach-Gesellschaft*, Hans David is of the opinion that *only* the one which Bach actually intended employed strict *real* imitation after five and a half measures, a viewpoint with which this author concurs.[17]

16 Callender (2012) proposes algorithms capable of producing canons with multiple realizations. However, his mostly two-voice examples not only display a questionable treatment of dissonance along with "astylistic" melodic lines, but also avoid the more traditional octave/dominant orientation in points of imitation.

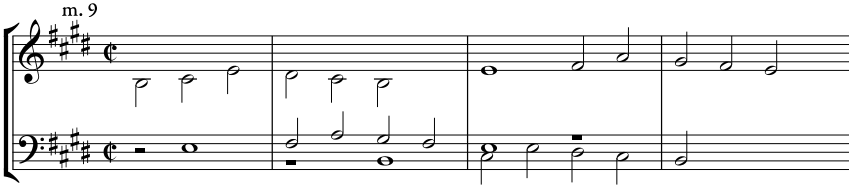
17 David (1945, 175).

William Renwick (1995) has posed the question that since "certain basic contrapuntal structures were reused over and over by generation after generation of composers," to what extent did they consider "the process of composition to be the art of reworking *existing* patterns?" Were they even "bequeathed from teacher to pupil in an oral or improvisational tradition?"[18] A survey of the canonic literature tends to confirm the recurrence of certain stretto paradigms which occur in various guises from the early Renaissance through the late Classical periods. In particular, the foregoing analyses have focused on three such prototypes, each of which will now be considered successively. The opening six-note segment of the *dux* of *Non nobis Domine* represents a variant of Renwick's Fugue Subject Paradigm 2, omitting the initial $\hat{3}$: $\hat{1}$ $\hat{2}$ ( ) $\hat{4}$ $\hat{3}$ $\hat{2}$ $\hat{1}$. The stretto of this subject in Fux's Fuga in C (Example 10a) may have provided the inspiration for J. K. Fischer's miniature E-major Fugue in his *Ariadne*

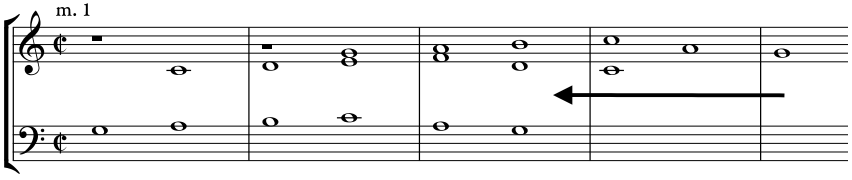
A. Fux: Fuga in C (*Gradus ad Parnassum*)



B. J.S. Bach: Fugue in E major (*Well-Tempered Clavier II*)



C. Palestrina: Pleni sunt caeli (*Missa ad fugam* 1567)

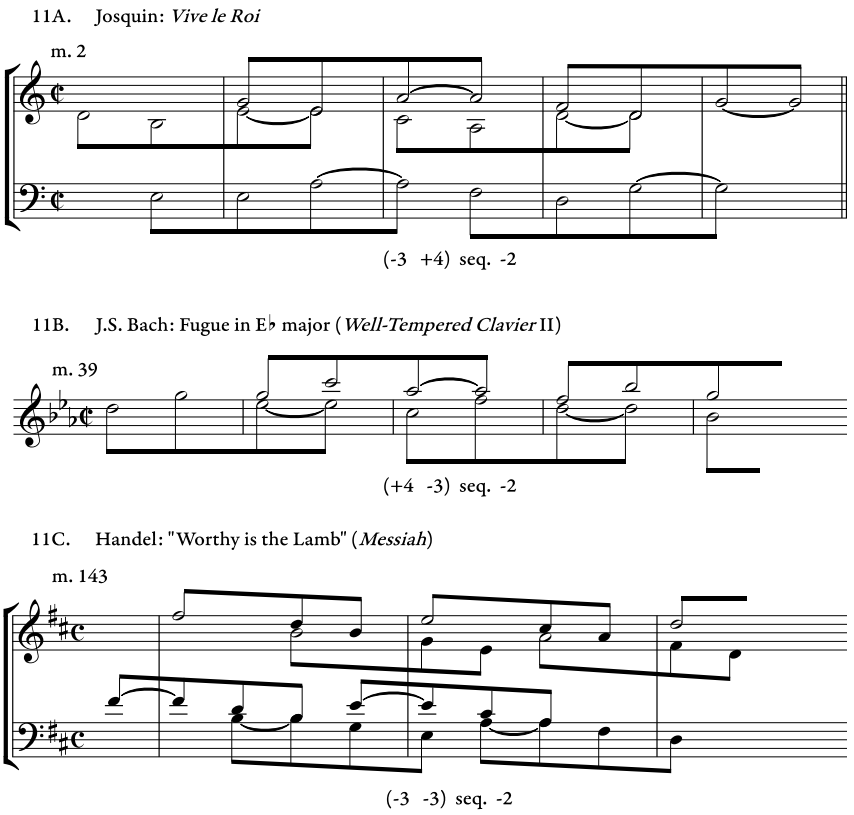


Example 10
Various stretti using the *Non nobis* paradigm.

18 Renwick (1995, 185).

Musica and its subsequent expansion in Bach's *Well Tempered Clavier II* (Example 10b). An interesting retrograde (!) version of the *Non nobis* segment occurs in Palestrina's "Pleni sunt caeli" (*Missa ad fugam*); see Example 10c.

Sequential treatment of the melodic cells in Stölzel's canon (Motive x = -3+4 and Motive y = -3-3, both of which lend themselves to multiple stretti) have been exploited by numerous composers. Josquin was already aware of the possibilities exhibited by the -3+4 motive, as his canonic tribute in Example 11a illustrates.[19] Bach on the other hand employs its inverted form (+4-3) as the basis for a two-voice episodic passage



Example 11
Various stretti using the -3+4 and -3-3 paradigms (shown in reduction).

19 Its "mother" pitch set [025] spawns several other permutations that are especially compatible to sequential stretti. This characteristic may originate from its interval vector, which features the most common intervals in the diatonic collection (M2, m3, and P4).

(Example 11b). Since a "cycle of fifths" progression is normally involved in a -3-3 sequence, it was frequently utilized by Baroque composers; the excerpt from Handel's *Messiah* (Example 11c) is a typical instance.

And then, there is Mozart. Aside from the celebrated Atwood studies in species counterpoint, the influence of Fux's pedagogical methodology sporadically appears in the initial subjects of his hybrid fugue/sonata movements, of which the Finales of the *Jupiter* Symphony K. 551[20] and the G-major "Haydn" Quartet K. 387 are among the most notable. Indeed, the opening scale degrees $\hat{1}\ \hat{2}\ \hat{4}\ \hat{3}$ of the former[21] duplicate those of the *dux* in *Non nobis Domine*,[22] which is not surprising since Mozart possessed a personal copy of that little piece which probably dated from his earlier canonic studies under Martini. It is also likely that he was acquainted with Marpurg's *Abhandlung* (1753-54), which may have paralleled his interest in Bach's contrapuntal technique about 1762. If so, he was doubtless aware of Stölzel's canon cited there. But does any tangible evidence exist to substantiate such a claim?

An examination of several passages from the "Jupiter" Finale raises the possibility that they may represent (conscious?) paraphrasing of Stölzel's canon.[23] Set in the dominant key of G major, the movement's secondary theme (mm. 80–83) appears to have been "lifted" directly out of Stölzel's *dux*, even to the extent of retaining the *identical* pcs (Example 12a). Mozart's subsequent expansion closely resembles the "Stölzel 1"realization. During the *tutti* passage that forms a portion of the exposition's closing section, he first isolates Motive x (-3+4), whose two-voice plagal stretto ascends by 3rds accompanied by a "filler" voice (Example 12b). As it turns to descend, Mozart then employs Stōlzel's original four-voice stretto by sequential thirds (Example 12c). If a possible correlation does exist between the two passages, it is a testimony to Mozart's genius that he was able to transform a pedantic little exercise into a symphonic pronouncement of powerful impact.

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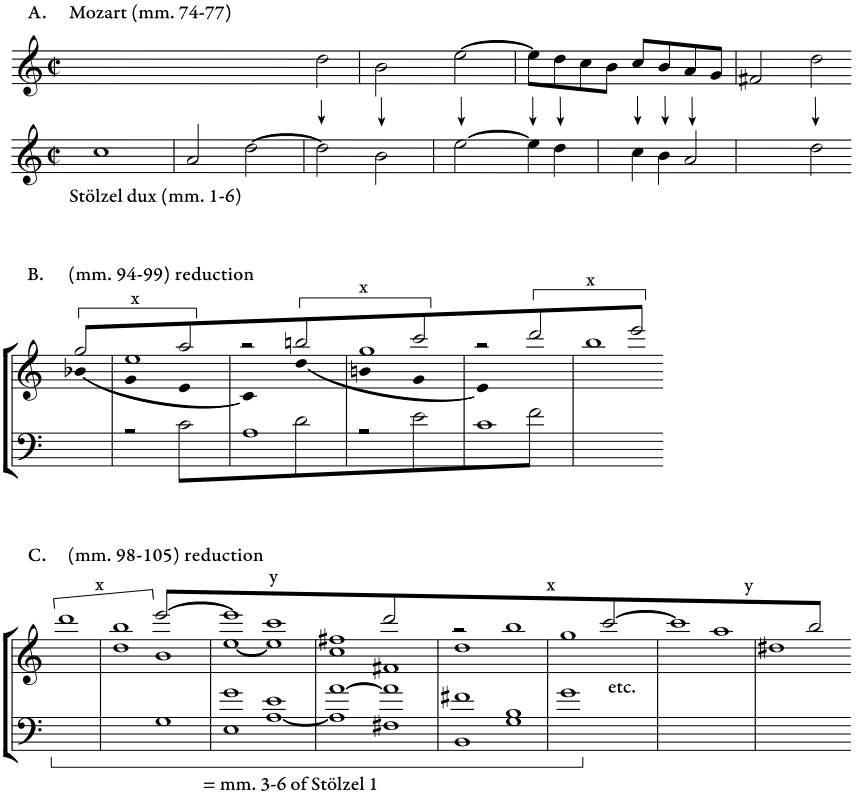
20 Gauldin (1991) explores the relation of the Finale's thematic material and contrapuntal combinations to strict species technique.

21 Derr (1985) cites other occurrences of this melodic cell in Mozart's output.

22 Although Mozart doubtless knew the little C-major fugue in *Parnassum* cited earlier, he refrained from exploiting stretto with this theme.

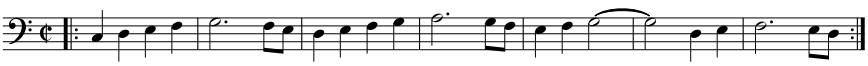
23 In his 1843 edition of Marpurg, Simon Sechter appended an analysis of the Finale to Mozart's "Jupiter" Symphony. While Sechter employs examples of "first species" reduction, he makes no reference of a possible relationship to Stölzel's canon.



Example 12
Comparisons of Stölzel's canon with passage from Mozart's "Jupiter" Finale.

We close this portion of the article with an additional conundrum from Stölzel, in which several realizations may be derived from its *dux* (Example 13).[24] Since all indications of pitch and temporal entrances have been intentionally deleted, this little piece assumes the nature of a *riddle stretto canon*, providing a bit of "homework" for the reader. A careful analysis of the completed solutions will reveal a close parallel to his previous polymorphic canon. However, caution is advised since several supposed realizations may contain harmonic problems, such as unexplained $^{6}_{4}$s.

24 This *dux* is quoted by Albrechtsberger in his *Grūndliche Anweistung zur Komposition* (1790), where he employs it to denote different means of pitch/temporal entries. There is no indication that he was aware of its polymorphic properties.



Example 13
A *dux* by Stölzel capable of several realizations.

Some Practical Suggestions For Composing Polymorphic Canons

It is obviously easier to point out the structural characteristics of a pre-existent polymorphic canon than to construct one from scratch. The following guidelines are intended to maneuver a perspective composer through the potential minefields of this task. Needless to say, the choice of either a Renaissance or tonal style necessitates a subsequent adherence to their respective compositional practices and norms.

1. It is better to restrict any initial attempts to three-voice settings, as the addition of a fourth part considerably increases the degree of difficulty.

2. The *dux* normally originates in the soprano or bass voice. The first note should be scale degree 1, since other choices may require tonal answers, whose intervallic adjustments are foreign to canonic technique.

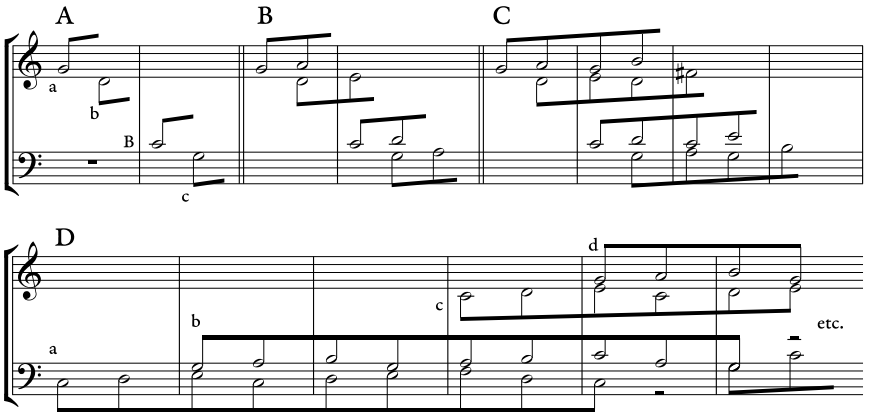
3. Try to confine the scale degrees of the *dux* within a major or minor hexachord (scale steps 1 through 6). The use of the leading tone sometimes results in cross relations or melodic/harmonic tritones.

4. The opening point of imitation in three-voice settings may conveniently employ those stretto models found in *Non nobis Domine*: either dominant or plagal orientation at T1 or T2, assuming the *dux* starts in the upper voice.[25] Lay out the initial pcs of the entries (*a*, *b*, *B* and *c*) in semi-breve reduction using a "dual" format (see one of the four possibilities as a model in Example 14a).

5. As each new pc is added to the original note of the *dux*, it is imitated in the succeeding voices. In Example 14b, G_4 is the first note, and A_4 has been selected for its second tone and subsequently distributed in the other parts, forming the HI of a fifth between a^2/b^1 and b^2/c^1. Disregard any harmonic dissonant intervals between *b*/*B*, since they will not appear in either of our eventual realizations.

25 If the canon begins in the bass, the resulting imitation indicators would tend to be "inverted": either plagal (P +4,+5 /T1,2) or dominant (P +5,+4/ T2,1).

6. This procedure is then continued in like manner, with the new G_4 in the dux (a^3) forming a third between a^3/b^2 and B^3/c^2 as well as a fifth between a^3/B^1 and b^3/c^1 (Example 14c). During the subsequent process, the choices for each new *dux* note will be become increasingly restrictive (a natural consequent of the "nature of the beast" with which we are dealing). Of course, other compositional formats are possible for the initial point of imitation of polymorphic canons. For instance, one may sketch a four-voice dominant-oriented canon (in first species) employing *lengthier* temporal distances between entries, as shown in Example 16d.[26] Provided that no harmonic $^6_4$s occur, four *different* embedded three-voice realizations may then be extracted: (*a*,*b*,*c*), (*b*,*c*,*d*), (*a*,*b*,*d*), and (*a*,*c*,*d*).[27] While the (*a*,*b*,*c*) canon retains its dominant orientation (P+5,+4/T2,4), the (*b*,*c*,*d*) version transposes to G major with a resultant plagal orientation (P+4,+5/T4,2).



Example 14
Suggested procedures for the initiation of a polymorphic three-voice canon.

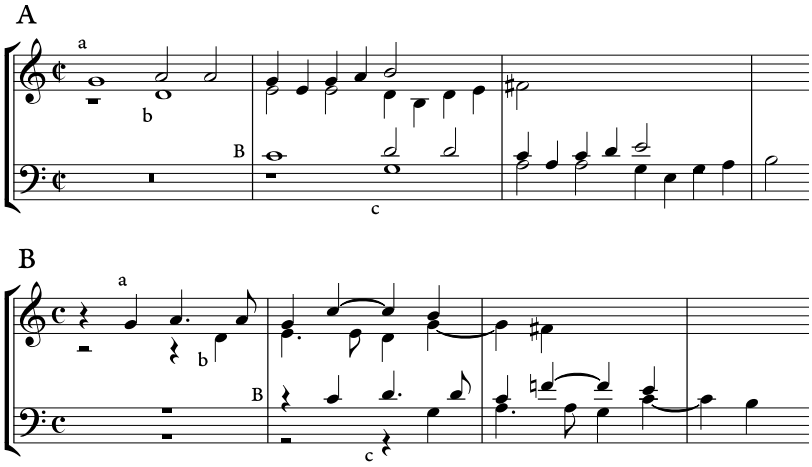
7. Several paths of continuation are available, depending on the overall "nature" of the canon desired. a. New pcs may be appended to the dux until an appropriate conclusion seems feasible, resulting in a kind of *through-*

26 Generally speaking, the lengthier temporal distances (T) between entries, the easier the construction of the first-species tonal framework, a situation which occurs more frequently in Baroque and Classical practice.

27 Altogether, there are a total of eight two- to four-voice solutions for this *dux*.

composed format; b. An opening "segment" of the *dux* (usually between six to eight pcs in species reduction) may be restated by diatonic sequence to produce a perpetual canon (as in *Non nobis*), in which case various "overlappings" must be navigated; and c. The *dux* may consist of two superimposed stretto segments built on different melodic cells, as was the case in "Stölzel 3." Other possible constructs depend on the discretion of the composer.

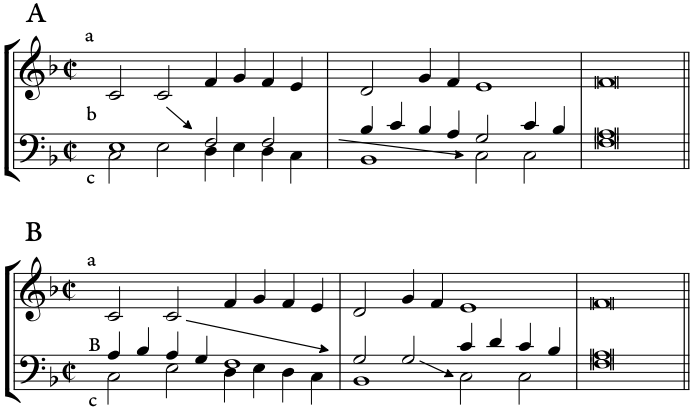
8. The elaboration of the first-species framework is left to the ingenuity of the composer. It may proceed concurrently with the framework or wait until the basic structure is complete. Based on Example 14c, Example 15a employs some passing motion in invoking a Renaissance style, while Example 15b assumes a more Baroque character with its suspensions. The F♯/F clash in the latter will obviously cancel out.



Example 15
Two elaborations of the framework in Example 14b.

9. A single issue remains: how to bring our composition to a satisfactory termination. Each of the two options is fraught with differing problems. In those "perpetual" situations where the canon cycles back to its initial point of imitation, the concluding pcs must overlap harmonically with those of their commencement, a procedure previously discussed in the *Non nobis* and Stölzel pieces. The alternative—that of establishing a standard cadential formula *within* the strict canonic confines—can prove a fiendishly difficult

task, since not only must those melodic scale degrees associated with a particular close be "anticipated" in the previous voices,[28] but *two* different versions of the cadence must be worked out to accommodate the *b* and *B* strands. The solutions for an authentic close appear in Example 16. In the first plagal imitation (Example 16a) the initial MI +4 moves successively to the middle voice before providing $\hat{5}$ to $\hat{1}$ in the bass for the cadence. In like manner, the rising third in m. 2 is also transferred, leaving the soprano to supply the remaining $\hat{7}$–$\hat{8}$ of the close. A similar process occurs in Example 16 as well, using dominant imitation.

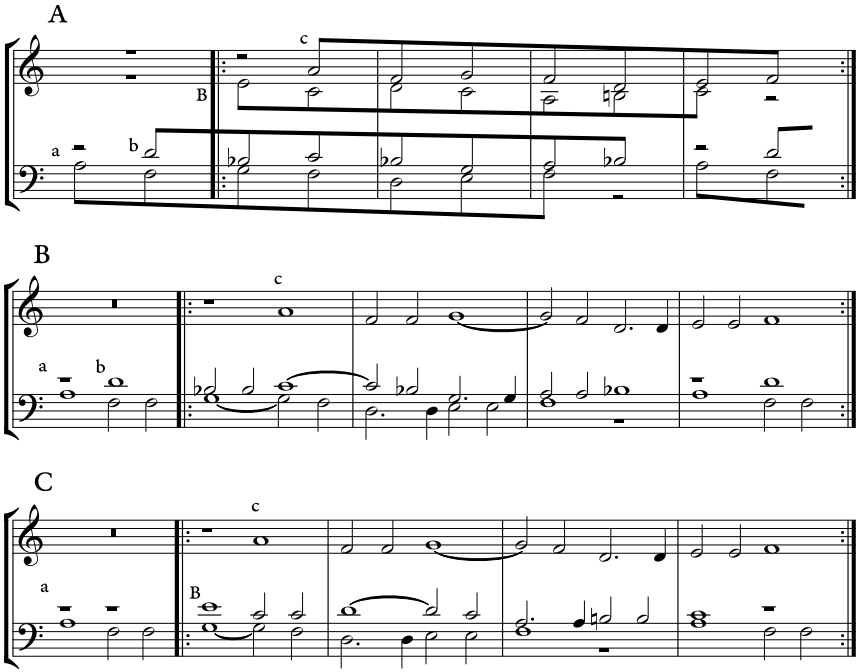


Example 16
The use of a standard cadential formula to terminate the canon.

Employing the above guidelines, Example 17 illustrates a pair of short three-voice perpetual canons based on the same *dux* in the bass voice. Example 17a lays out their "dual" first-species framework. Note that the B natural/B flat clash in m. 4 will not appear when the two realizations are separated in Examples 17b (plagal) and 17c (dominant).[29] Their elaborations feature suspensions and repeated notes but no passing tones, some of which would produce parallel fifths.

28 This procedure for asymmetrical canons is briefly discussed in Gauldin (1996, 47–50).

29 The imitation is real throughout. While the *dux* (*a*) exploits the natural hexachord, (*b*) and (*B*) employ the moll and durum, respectively.



Example 17
A brief original *dux* with two three-voice realizations.

In conclusion, this study could be considered as an addendum to the section of Prout's *Double Counterpoint and Canon* denoted to that particular polyphonic idiom. Although he includes both of the canons examined here, he only cites various realizations of the Stölzel piece.[30] Therefore, the detailed analyses, "homework," and compositional guidelines in this article provide a more pedagogically oriented supplement to Prout's discussion of the polymorphic genre.

One might ask where the preceding material would occupy in a curriculum. Although it is hardly appropriate for a beginning course in tonal counterpoint, it could be assigned as auxiliary reading in a graduate class, or function as a special topics seminar, or even form the subject for individual research or a possible thesis. A survey of the preceding analyses suggest several questions of a more general nature which could provide viable topics for future research. Are there only a limited number of

30 Curiously enough, Prout fails to mention that *Non nobis Domine* (incorrectly attributed to William Byrd on p. 208) is polymorphic.

melodic paradigms which are compatible to the construction of polymorphic canons? Would comparisons of those paradigms in different stretto sections of the same fugue (such as those listed by William Renwick) provide clues in the regard? Excluding those canons extended by cyclical transposition, is there a "practical limit" to the duration of polymorphic canons? It there a "parallel universe" of these canons which eschew imitations by perfect intervals in favor of thirds or seconds? Are chromatic idioms to be completely avoided, or do a small group of such feasible melodic paradigms exist?

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